What is Digital Signal Processing?

To understand what is Digital Signal Processing (DSP) let's examine what does each of its words mean.

Signal is any physical quantity that carries information. Processing is a series of steps or operations to achieve a particular end. It is easy to see that Signal Processing is used everywhere to extract information from signals or to convert information-carrying signals from one form to another. For example, our brain and ears take input speech signals, and then process and convert them into meaningful words. Finally, the word Digital in Digital Signal Processing means that the process is done by computers, microprocessor, or logic circuits.

The field DSP has expanded significantly over that last few decades as a result of rapid developments in computer technology and integrated-circuit fabrication. Consequently, DSP has played an increasingly important role in a wide range of disciplines in science and technology. Research and development in DSP are driving advancements in many high-tech areas including telecommunications, multimedia, medical and scientific imaging, and human-computer interaction.

Concepts in Digital Signal Processing

The two main characters in DSP are signals and systems. A signal is defined as any physical quantity that varies with one or more independent variables such as time (one-dimensional signal), or space (2-D or 3-D signal). Signals exist in several types. In the real-world, most of signals are continuous-time (analog signals) those have values continuously at every value of time. To be processed by a computer, a continuous-time signal has to be first sampled in time into a discrete-time signal so that its values at a discrete set of time instants can be stored in computer memory locations. Furthermore, in order to be processed by logic circuits, these signal values have to be quantized in to a set of discrete values, and the final coded result is called a digital signal. The terms discrete-time signal and digital signal can be used interchangeability to define two different formats (Fig. 1).

In signal processing, a system is defined as a process coder whose input and output are signals (Fig. 2).

Signals Represent Information

Whether analog or digital, information is represented by the fundamental quantity in electrical engineering: the signal. Stated in mathematical terms, a signal is merely a function. Analog signals are continuous-valued; digital signals are discrete-valued. The independent variable of the signal could be time (speech), space (images), or the integers (denoting the sequencing of letters and numbers in the football score).

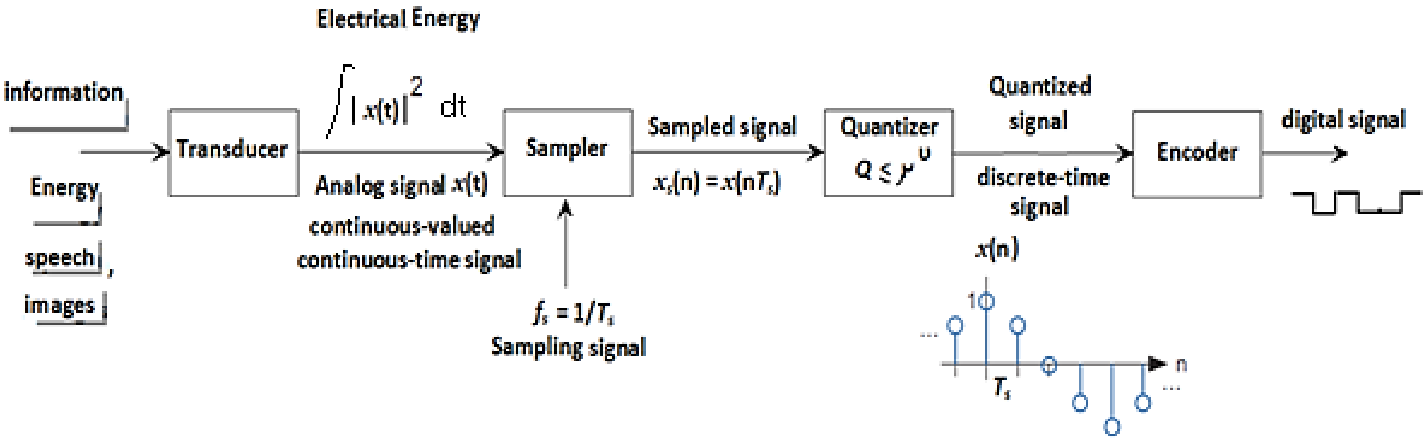

Electrical Energy
information
Energy
speech
images
Transducer
Analog signal x(t)
continuous-valued
continuous-time signal
Sampler
Sampled signal
Sampling signal
Quantizer
Quantized
signal
discrete-time
signal
x(n)
Encoder
digital signal

Fig. 1

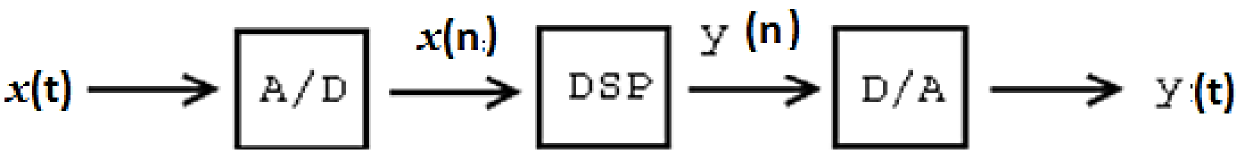

x(t)
A/D
x(n)
DSP
y (n)
D/A
y (t)

Fig. 2

Sampling

Why sample? Sampling is the necessary fundament for all digital signal processing and communication. Sampling can be defined as the process of measuring an analog signal at distinct points. Digital representation of analog signals offers advantages in terms of

1-Robustness towards noise, meaning we can send more bits/s.
2-Use of flexible processing equipment, in particular the computer.
3-More reliable processing equipment.
4-Easier to adapt complex algorithms.

Claude Shannon has been called the father of information theory, mainly due to his landmark papers on the "Mathematical theory of communication". Harry Nyquist was the first to state the sampling theorem in 1928, but it was not proven until Shannon proved it 21 years later in the paper "Communications in the presence of noise".

The following notations will be used: Original analog signal x(t), Sampling frequency f_s , Sampling interval T_s (Note that: $f_s = 1/T_s$), Sampled signal x_s(n). (Note that $x_s(n) = x(nT_s)$), Analogue angular frequency Ω , and Digital angular frequency ω (Note that: $\omega = \Omega T_s$).

The Sampling Theorem

[[When sampling an analog signal the sampling frequency must be greater than twice the highest frequency component of the analog signal to be able to reconstruct the original signal from the sampled version]].

The process of sampling

We start with an analog signal. This can for example be the sound coming from your stereo at home or your friend talking. The signal is then sampled uniformly. Uniform sampling implies that we sample every T_s seconds. In Fig. 3, we see an analog signal. The analog signal has been sampled at times $t = nT_s$.

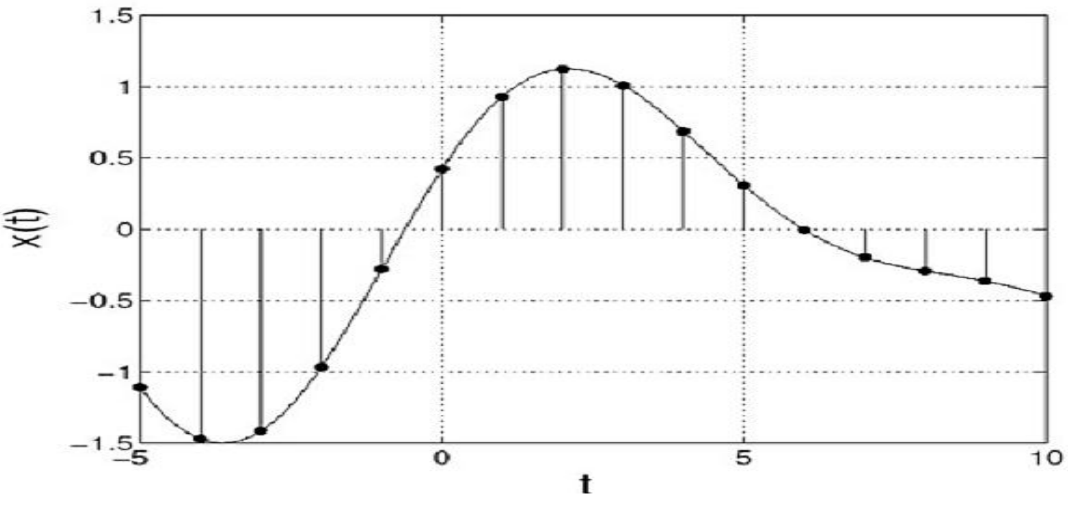


Fig. 3

In signal processing it is often more convenient and easier to work in the frequency domain. So let's look at the signal in frequency domain, Fig. 4 . For illustration purposes we take the frequency content of the signal as a triangle. (If you Fourier transform the signal in Fig. 3 you will not get such a nice triangle.)

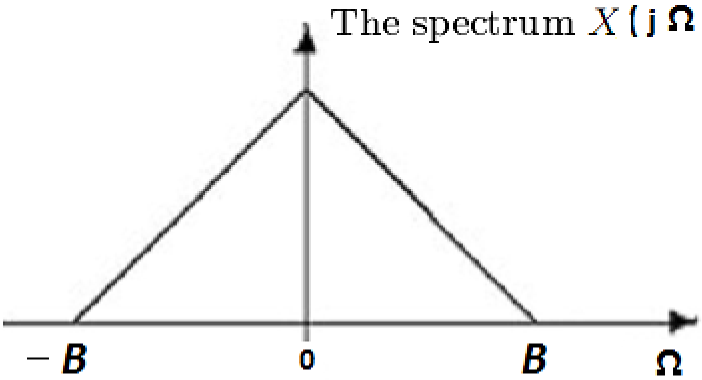


Fig. 4

Sampling fast enough

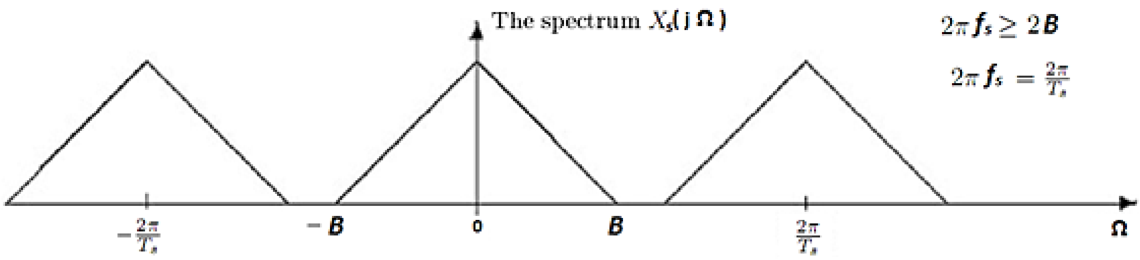


Fig. 5

From Fig. 5, and according to the sample theorem, an aliasing-free condition appears. So, we are able to reconstruct the original signal exactly. How can we do this? will be explored further down under reconstruction. But first we will take a look at what happens when we sample too slowly.

Sampling too slowly

We will get overlap between the repeated spectra, see Fig. 6. The resulting spectra is the sum of these. This overlap gives rise to the concept of aliasing.

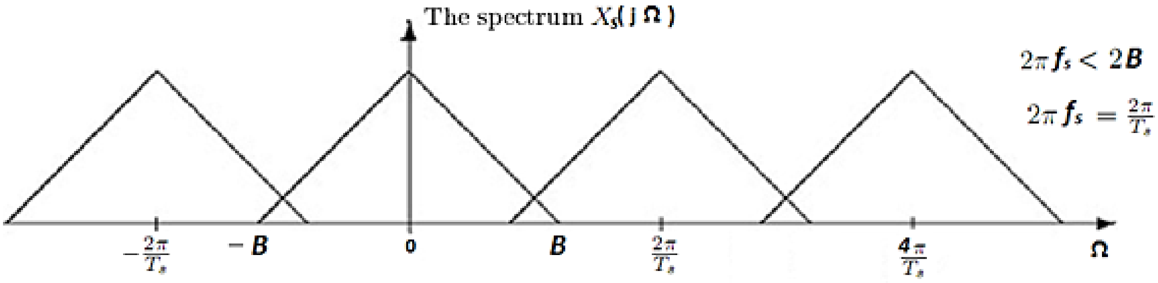


Fig. 6

To avoid aliasing we have to sample fast enough. But if we can't sample fast enough (possibly due to costs) we can include an Anti-Aliasing filter. This will not able us to get an exact reconstruction but can still be a good solution.

Note: Typically a low-pass filter that is applied before sampling to ensure that no components with frequencies greater than half the sample frequency remain.

Reconstruction

We want to recover the original signal, but the question is how?

The Answer: By using a simple reconstruction process. To achieve this we have to remove all the extra components generated in the sampling process. To remove the extra components we apply an ideal analog low-pass filter as shown in Fig. 7. As we see the ideal filter is rectangular in the frequency domain. A rectangle in the frequency domain corresponds to a sinc function in time domain (and vice versa).

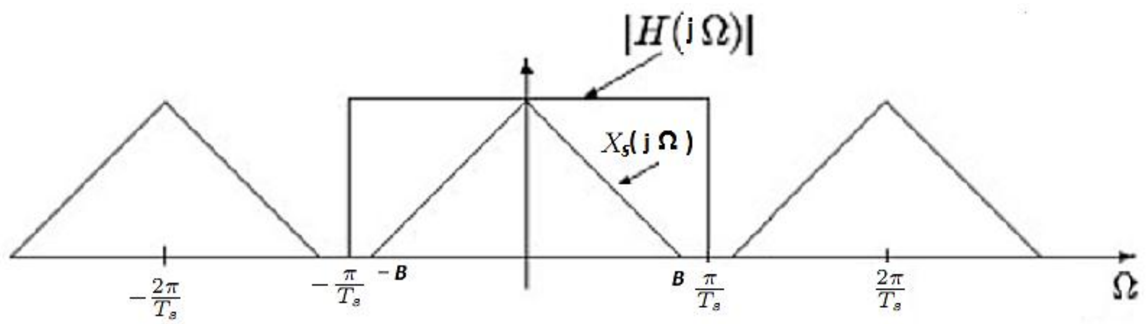
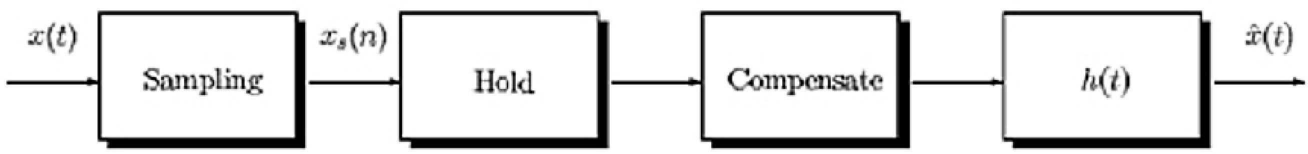


Fig. 7

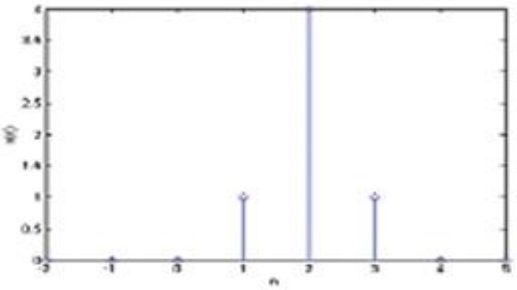


Discrete-Time Signals

- A Discrete-time signal $x(n)$ is a function of an independent integer variable *n*. The signal $x(n)$ is not defined for non-integer values of *n*.

We can represent a discrete-time signal in different ways;

1. Graphical representation Such as



Discrete-Time Signals

2. Functional representation

Such as

$$x(n)=\begin{cases}1, & \text{for } n=1,3\\ 4, & \text{for } n=2\\ 0, & \text{elsewhere}\end{cases}$$

$$x(n)=(0.5)^{n}$$

3. Tabular representation

Such as

n	...	-2	-1	0	1	2	3	4	5	...
$x(n)$	...	0	0	0	1	4	1	0	0	...

4. Sequential (Vector) representation

Such as

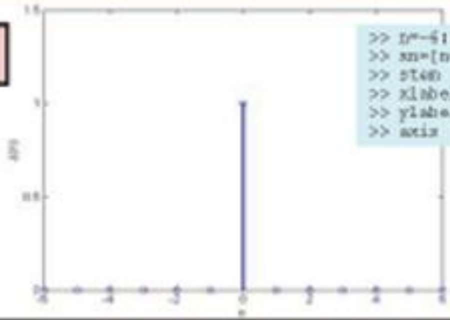
$$x(n)=[\ldots 0,0,0,1,\underset{\uparrow}{4},1,1,0,0,\ldots]$$

The time origin (n=0) is indicated by the symbol ↑.

Some Elementary Discrete-Time Signals

1. The unit sample sequence:

$$\delta(n)=\begin{cases}1, & \text{for } n=0,\\ 0, & \text{for } n\neq 0\end{cases}$$

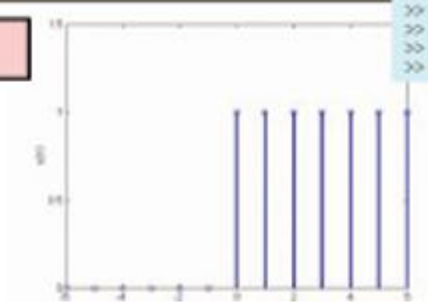


```
>> n=-6:6;
>> xn=[n==0];
>> stem (n,xn)
>> xlabel('n')
>> ylabel('\delta(n)')
>> axis ([-6 6 0 1.5])
```

The unit sample sequence is often referred to as a discrete-time impulse or an impulse.

2. The unit step signal :

$$u(n)=\begin{cases}1, & \text{for } n\geq 0,\\ 0, & \text{for } n<0\end{cases}$$



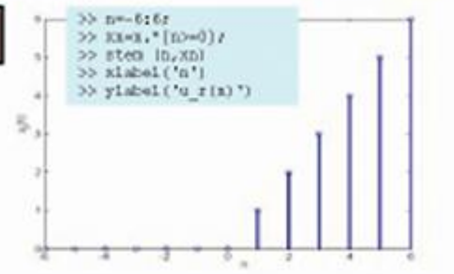
```
>> n=-6:6;
>> xn=[n>=0];
>> stem (n,xn)
>> xlabel('n')
>> ylabel('u(n)')
>> axis ([-6 6 0 1.5])
```

$note$: $u(n)=\sum_{k=0}^{\infty}\delta(n-k)$, $also$ $\delta(n)=u(n)-u(n-1)$

Some Elementary Discrete-Time Signals

3. The unit ramp signal :

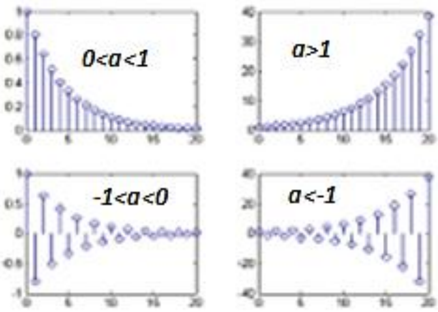
$$u_r(n) = \begin{cases} n, & \text{for } n \geq 0, \\ 0, & \text{for } n < 0 \end{cases}$$



4. The exponential signal :

$$x(n) = a^n \quad \text{for } n \geq 0$$

$$x(n) = a^n \, u(n)$$



If *a* is real, then $x(n)$ is a real signal.

Some Elementary Discrete-Time Signals

If a is complex, $x(n)$ can be expressed as

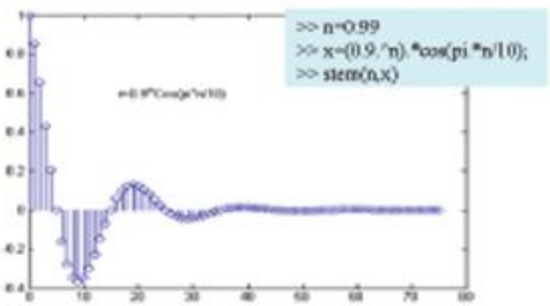
$$x(n) = a^n = \left(re^{j\theta}\right)^n = r^n e^{j\theta n}$$
$$= r^n(\cos\theta n + j\sin\theta n)$$

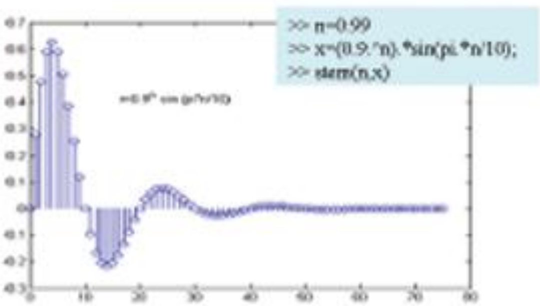
The real part is

$$x_R(n) = r^n \cos\theta n$$

The imaginary part

$$x_R(n) = r^n \sin\theta n$$





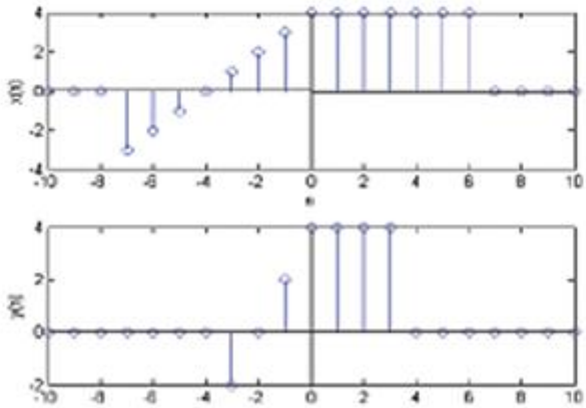
Simple Manipulation of $x(n)$

Example:

$x(n)$=[0,0,0,-3,-2,-1,0,1,2,3,4,4,4,4,4,4,4,0,0,0]
↑

$y(n) = x(2n)$ Find $y(n)$.

$y(0) = x(0),\ y(-1) = x(-2),\ y(1) = x(2),\ y(-2) = x(-4),\ y(2) = x(4)$



Classification of Discrete-Time Signal

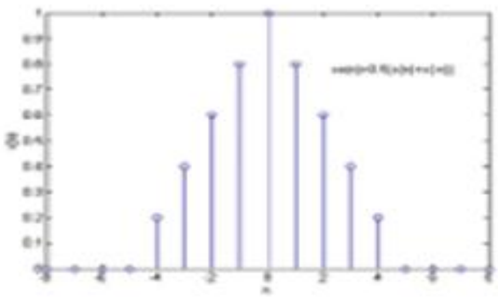
Symmetric (even) and anti-symmetric (odd) signals

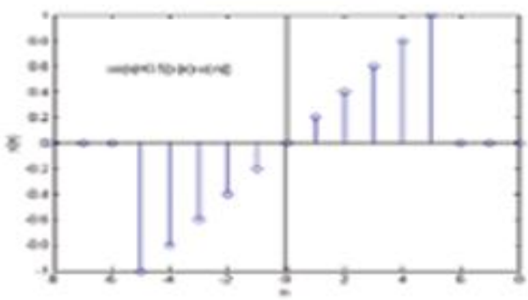
A real-valued signal $x(n)$ is called symmetric (even) if

$$x(-n) = x(n)$$

A signal $x(n)$ is called anti-symmetric (odd) if

$$x(-n) = -x(n)$$

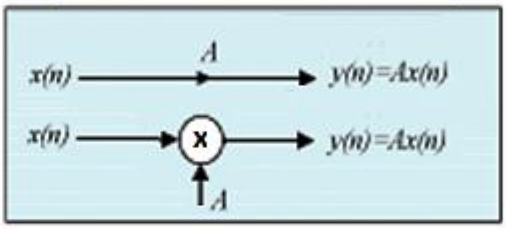




Addition, Multiplication, and Scaling of Sequences

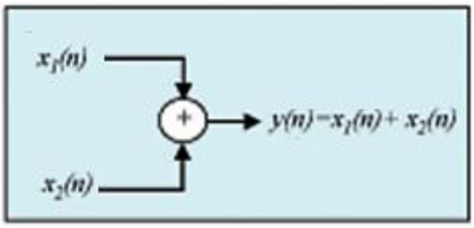
Amplitude Scaling: (A Constant Multiplier)

$$y(n) = Ax(n), \quad -\infty < n < \infty$$



Addition of two signals (An Adder)

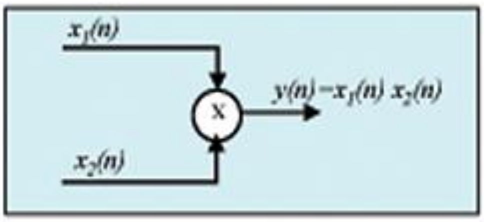
$$y(n) = x_1(n) + x_2(n), \quad -\infty < n < \infty$$



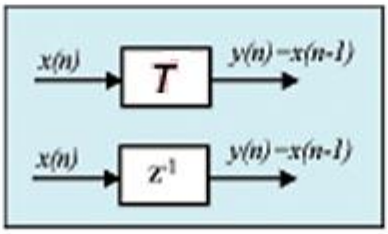
Addition, Multiplication, and Scaling of Sequences

The product of two signals (A signal Multiplier)

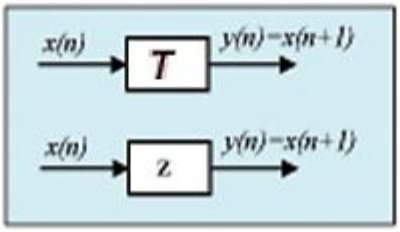
$$y(n) = x_1(n)x_2(n), \quad -\infty < n < \infty$$



A unit delay element

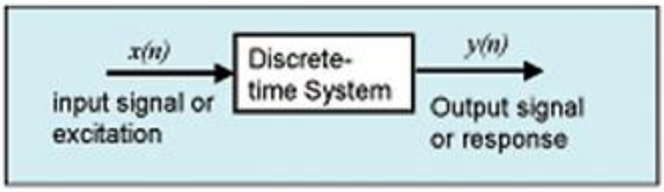


A unit advance element



Input-Output Description of Systems

The relation between the input and output signals are known input-output relationship



Mathematical representation of the transformation is

$$y(n) = \mathrm{T}[x(n)]$$

T denotes the transformation. In general input-output relationship can be also shown as

$$x(n) \xrightarrow{\mathrm{T}} y(n)$$

Input-Output Description of Systems

Example: The input signal is

$$x(n) = \begin{cases} |n| & -3 \le n \le 3 \\ 0 & \text{otherwise} \end{cases}$$

a $y(n) = x(n)$

b $y(n) = x(n-1)$

c $y(n) = x(n+1)$

d $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

e $y(n) = \max[x(n+1), x(n), x(n-1)]$

f $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \ldots$

Solution

$$x(n) = [\ldots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \ldots]$$

a $y(n) = x(n)$ $\quad y(n) = [\ldots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \ldots]$

b $y(n) = x(n-1)$ $\quad y(n) = [\ldots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \ldots]$

Input-Output Description of Systems

Solution (cont)

c $y(n) = x(n+1)$ $\quad y(n) = [\ldots, 0, 3, 2, 1, 0, 1, 2, 3, 0, \ldots]$

d $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$

$$y(0) = \frac{1}{3}[x(1) + x(0) + x(-1)] = \frac{1}{3}[1 + 0 + 1] = \frac{2}{3}$$

$$y(-1) = \frac{1}{3}[x(0) + x(-1) + x(-2)] = \frac{1}{3}[0 + 1 + 2] = 1$$

.....

$$y(n) = [\ldots, 0, 1, \frac{5}{3}, 2, 1, \frac{2}{3}, 1, 2, \frac{5}{3}, 1, 0, \ldots]$$

f $y(n) = \max[x(n+1), x(n), x(n-1)]$

$$y(0) = \max[x(1), x(0), x(-1)] = \max[1, 0, 1] = 1$$

$$y(-1) = \max[x(0), x(-1), x(-2)] = \max[0, 1, 2] = 2$$

$$y(1) = \max[x(2), x(1), x(0)] = \max[2, 1, 0] = 2$$

$$y(n) = [0, 3, 3, 3, 2, 1, 2, 3, 3, 3, 0, \ldots]$$

Input-Output Description of Systems

Solution (cont)

e

$$y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \ldots$$

This system is called an accumulator

$$y(0) = \sum_{k=-\infty}^{0} x(k) = x(0) + x(-1) + x(-2) + x(-3)$$
$$= 0 + 1 + 2 + 3 = 6$$

$$y(1) = \sum_{k=-\infty}^{1} x(k) = x(1) + x(0) + x(-1) + x(-2) + x(-3)$$
$$= 1 + 0 + 1 + 2 + 3 = 7$$

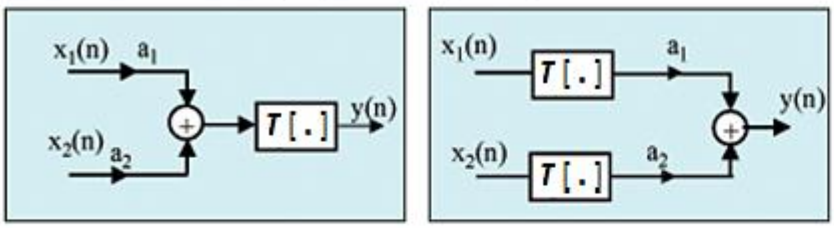
$$y(n) = [\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 12, \ldots]$$

Classification of Discrete-Time Systems

Linear System: A system is linear if and only if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

for any arbitrary $x_1(n)$ and $x_2(n)$, and any arbitrary constant a_1 and a_2. It satisfies the superposition principle.



If the input is $x_1(n)$, *the output will be* $y_1(n)$
if the input is $x_2(n)$, *the output will be* $y_2(n)$

if the input is $[x_1(n) + x_2(n)]$, *the output will be* $y(n)$

if $y(n) = [y_1(n) + y_2(n)]$, *then the system is linear.*

Example 1: $y(n) = e^{-x(n)}$

solution: $for\ y_1(n) = e^{-x_1(n)}$

$for\ y_2(n) = e^{-x_2(n)}$

$for[x_1(n) + x_2(n)],\quad y(n) = e^{-[x_1(n)+x_2(n)]}$

$[y_1(n) + y_2(n)] = e^{-x_1(n)} + e^{-x_2(n)}$

$\neq e^{-[x_1(n)+x_2(n)]}$ It is non − linear system

Classification of Discrete-Time Systems

Time-invariant Systems:

A system is called time-invariant if its input-output characteristics do not change with time.

a- shift the input $x(n-n_0)$

b- shift the output $y(n-n_0)$

if $y(n-n_0) = y(n)$ then the system is time-invariant.

Example 2 : $y(n)=n\,x(n-1)$

solution: Shift the input, $y(n)=n\,x(n-n_0-1)$

shift the output, $y(n-n_0)=(n-n_0)\ x(n-n_0-1)$

since $y(n-n_0) \neq y(n)$, then the system is time-variant.

Note: If the system (time-invariant & linear), it is called Linear Time Invariant Systems (LTI)

LTI system can easily be characterized by its output response $h(n)$ to the input $\delta(n)$. The input-output relationship can then be given by CONVOLUTION

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Classification of Discrete-Time Systems

Causal Sequence:

A Sequence x(n) is called *causal* if x(n)=0 , *for* n<0

Causal Systems:

Difference Equation (D.E.) Representation

A system is called *causal* if the output of the system at any time n depends only on present and past inputs, but not depends on future inputs.

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots]$$

An LTI System with impulse response h(n) is called *causal* if h(n)=0 , *for* n<0

If the system does not satisfy any of those definitions, it is called *noncausal*.

Example 3: y(n)= x(n+1)

At n = 0, y(0) = x(1), then the system is non-causal (anti-causal).

Example 4: $h(\mathrm{n}) = 0.5^n u(\mathrm{n})$

Since $h(\mathrm{n}) = 0$, for $\mathrm{n} < 0$, then it is a causal system

Classification of Discrete-Time Systems

bounded Sequence:

A **Sequence** $x(n)$ is called bounded if $|x(n)| \leq M_x < \infty$

Stable Systems:

Difference Equation (D.E.) Representation

An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces a bounded output.

$|x(n)| \leq M_x < \infty$ $\qquad$ $|y(n)| \leq M_y < \infty$

for all n. M_x and M_x are some finite numbers

An **LTI System with** $h(n)$ is (BIBO) stable if $\sum |h(n)| = M < \infty$

If the system does not satisfy any of those definitions, it is called unstable.

<u>*Example*</u> 5: y(n)= n x(n-1)

As n $\rightarrow \infty$, y(n) $\rightarrow \infty$, then the system is unstable.

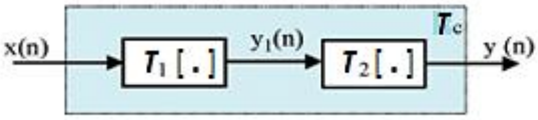
<u>*Example*</u> 6: $h(\mathrm{n}) = ({}^{1}/_{4})^{\mathrm{n}} u(\mathrm{n})$

Since $\sum_{k=0}^{\infty} |h(\mathrm{n})| < \infty$, it is a stable system

Interconnection of Discrete-Time Systems

Discrete-time systems can be interconnected to form larger systems. They can be interconnected in serial or parallel.

In serial interconnection



$y_1(n) = T_1[x(n)]$

$$y(n) = T_2[y_1(n)] = T_2[T_1[x(n)]]$$

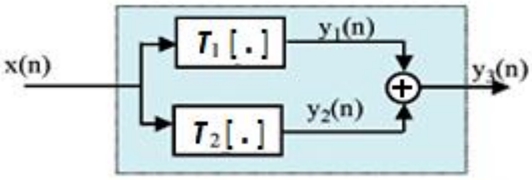
If we combine T_1 and T_2 to , then $y(n) = T_c[x(n)]$

If the systems T_1 and T_2 are linear and time invariant $T_1 T_2 = T_2 T_1$

otherwise $T_1 T_2 \neq T_2 T_1$

Interconnection of Discrete-Time Systems

In parallel interconnection



$$\begin{aligned} y_3(n) &= y_1(n) + y_2(n) \\ &= T_1[x(n)] + T_2[x(n)] \\ &= (T_1 + T_2)[x(n)] \\ &= T_p[x(n)] \end{aligned}$$

where $T_p = T_1 + T_2$

Convolution

The response of the system to x(n) is

$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right] = \sum_{k=-\infty}^{\infty} x(k)T[\delta(n-k)]$$

The response of the LTI system to the unit sample sequence is denoted as $h(n)$, and it is called the impulse response of a linear time invariant system

$$h(n) = T[\delta(n)]$$

So the output sequence *y(n)* is found as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

This is called convolution

The response y(n) at $n=n_0$ is

$$y(n_0) = \sum_{k=-\infty}^{\infty} x(k)h(n_0-k)$$

The process of computing the convolution between *x(n)* and *h(k)* involves the following steps.

1. *Folding*. Fold $h(k)$ about $k=0$ to obtain $h(-k)$
2. *Shifting*. Shift $h(-k)$ by n_0 to the right (left) if n_0 is positive (negative), to obtain $h(n_0-k)$.
3. *Multiplication*. Multiply $x(k)$ by $h(n_0-k)$ to obtain the product sequence
4. *Summation*. Sum all the values of the product sequence to obtain the value of the output at time $n=n_0$.
5. Step 2 through 4 must be repeated, for all possible time shifts .

$$-\infty < n < \infty$$

Methods of Convolution

1- Graphical method.

2- Table-look up method.

3- Vector-by-matrix method.

4- Add-overlap method.

5- Analytical method.

1- Graphical Method

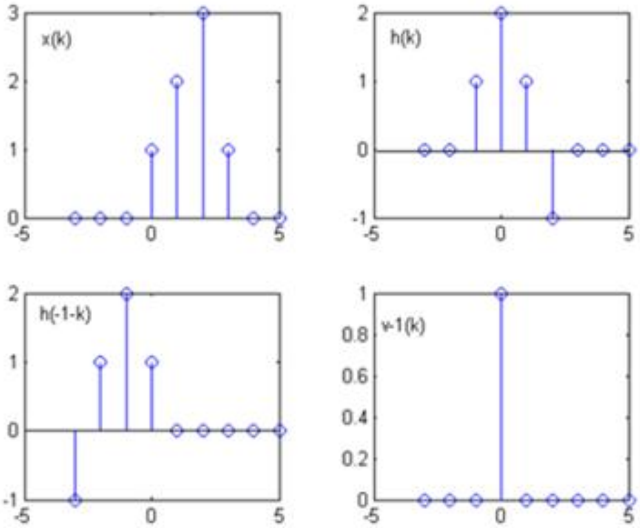
The impulse response of a linear time invariant system is $h(n) = \left[1, \underset{\uparrow}{2}, 1, -1\right]$

The input signal is $x(n) = \left[1, 2, 3, 1\right]$

Solution:

The output y(n) at n=-1 is $y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$

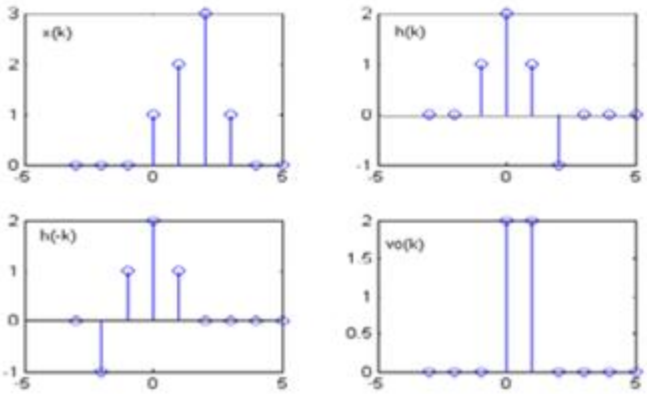
and the product sequence $v_{-1}(k) = x(k)h(-1-k)$



$$y(-1) = \sum_{k=-\infty}^{\infty} v_{-1}(k) = 1$$

The output at n=0 is $y(0)=\sum_{k=-\infty}^{\infty} x(k)h(-k)$

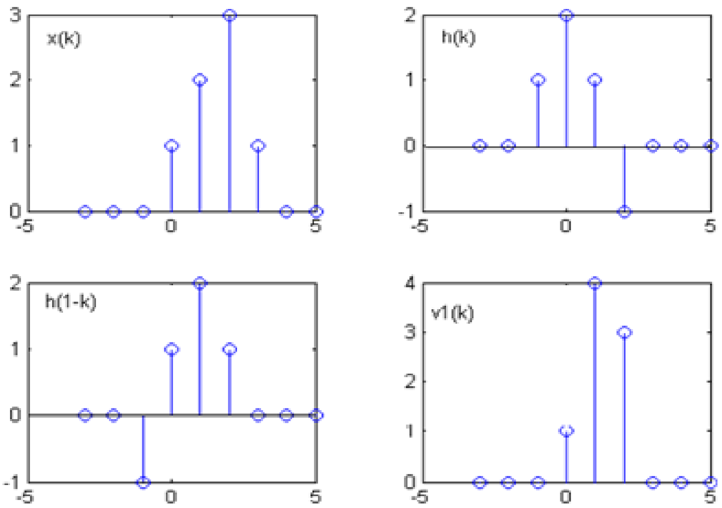
and the product sequence $v_0(k)=x(k)h(-k)$



$$y(0)=\sum_{k=-\infty}^{\infty} v_0(k)=2+2=4$$

Similarly, The output at n=1 is $y(1)=\sum_{k=-\infty}^{\infty} x(k)h(1-k)$

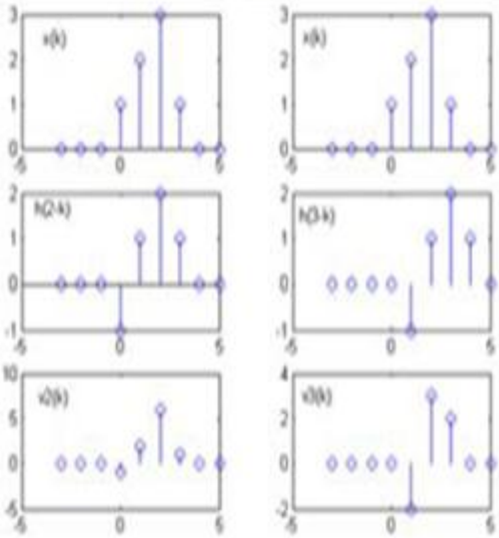
and the product sequence $v_1(k)=x(k)h(1-k)$



$$y(1)=\sum_{k=-\infty}^{\infty} v_1(k)=1+4+3=8$$

Solution (cont)

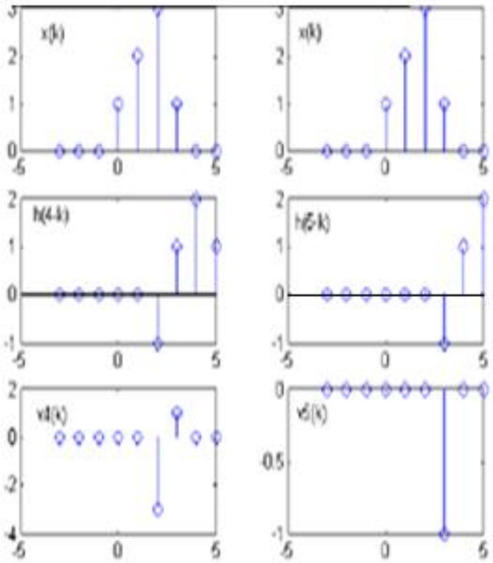
Let's find y(n) at n=2, and n=3



$$y(2)=\sum_{k=-\infty}^{\infty} v_2(k) = -1+2+6+1=8$$

$$y(3)=\sum_{k=-\infty}^{\infty} v_3(k) = -2+3+2=3$$

Let's find y(n) at n=4, and n=5



$$y(4)=\sum_{k=-\infty}^{\infty} v_4(k) = -3+1=-2$$

$$y(5)=\sum_{k=-\infty}^{\infty} v_5(k) = 1=1$$

$$y(n)=0,\ n\geq 6$$

The entire system response to x(n) is

$$y(n)=[\ldots,0,0,1,\underset{\uparrow}{4},8,8,3,-2,-1,0,0,\ldots]$$

Properties of Convolution

We show the convolution operation with asterisk

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Commutative Law

$$x(n) * h(n) = h(n) * x(n)$$

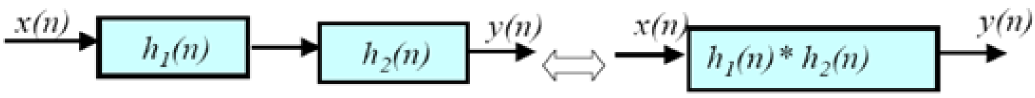
which means

$$\sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

Associative Law

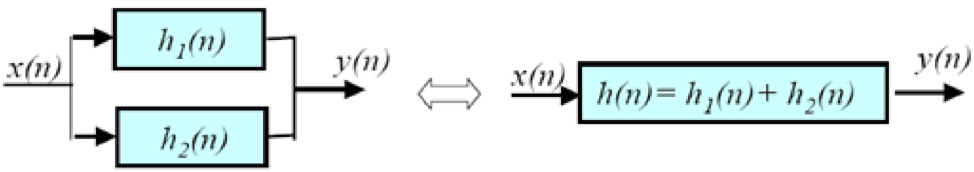
$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

$h(n) = h_1(n) * h_2(n)$ and $y(n) = x(n) * h(n)$



Distributed Law

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$



$h(n) = h_1(n) + h_2(n)$ and $y(n) = x(n) * h(n)$

2- Table-look up Method h(n)

$$x(n) = [1 \quad 2 \quad 3 \quad 1]\ ,\ h(n) = [\underset{\uparrow}{1} \quad 2 \quad 1 \quad -1\ ]$$

$find\ y(n)?$

x(n) / h(n)	1	2	3	1
1	1	2	3	1
→ 2	2	4	6	2
1	1	2	3	1
-1	-1	-2	-3	-1

$$y(n) = [\,\underset{\uparrow}{1} \quad 4 \quad 8 \quad 8 \quad 3 \ -2 \ -1]$$

3- Vector-by-matrix Method

$$x(n) = [\underset{\uparrow}{1} \quad 2 \quad 3 \quad 1]\ ,\ h(n) = [1 \quad 2 \quad 1 \quad -1\ ]$$

$find\ y(n)?$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 1 & 3 & 2 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 8 \\ 8 \\ 3 \\ -2 \\ -1 \end{bmatrix}$$

4- Add-overlap Method

$$x(n) = [1 \quad 2 \quad 1\ ]\ , \quad h(n) = [1\ -1 \quad 2 \quad 1 \quad 2\ -1 \quad 1 \quad 3 \quad 1\ ]$$

	1	2	1
1	1	2	1
-1	-1	-2	-1
2	2	4	2

$$y_1(n) = [\,1 \quad 1 \quad 1 \quad 3 \quad 2]$$

	1	2	1
1	1	2	1
2	2	4	2
-1	-1	-2	-1

$$y_2(n) = [\,1 \quad 4 \quad 4 \quad 0 \quad -1]$$

	1	2	1
1	1	2	1
3	3	6	3
1	1	2	1

$$y_3(n) = [\,1 \quad 5 \quad 8 \quad 5 \quad 1]$$

$$y_1(n) = \ [\,1 \quad 1 \quad 1 \quad 3 \quad 2\,]$$

$$Shifted\ y_2(n) = [\,0 \quad 0 \quad 0 \quad 1 \quad 4 \quad 4 \quad 0 \quad -1]$$

$$Shifted\ y_3(n) = [\,0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 5 \quad 8 \quad 5 \quad 1]$$

$$y(n) = \ y_1(n) + \ Shifted\ y_2(n) + \ Shifted\ y_3(n)$$

$$So, \qquad y(n) = [\ 1 \quad 1 \quad 1 \quad 4 \quad 6 \quad 4 \quad 1 \quad 4 \quad 8 \quad 5 \quad 1]$$

5- Analytical Method

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$postion \quad of \quad N_1 \le x(n) \le N_2$

$postion \quad of \quad M_1 \le h(n) \le M_2$

$y(n) = \sum_{k=k_l}^{k_u} x(k)\, h(n-k)$

How to calculate k_u and k_l ?

$for \quad x(n), \quad N_1 \le k \le N_2$

$for \quad h(n), \quad M_1 \le n-k \le M_2$

$or \quad M_1 - n \le -k \le M_2 - n$

$i.e, \quad n - M_2 \le k \le n - M_1$

$\therefore \quad k_l = max\,\{N_1, n - M_2\}$

and $k_u = min\{N_2, n - M_1\}$

Example: $Find \ \ y(n) \quad if \quad x(n) = u(n) - u(n-10) \quad and \quad h(n) = \left(\frac{1}{2}\right)^n u(n)$

Solution:-

N_1=0 , N_2=9

M_1=0 M_2=∞

$$k_l = max\{N_1, n - M_2\} = max\{0, n - \infty\} = 0$$

$$k_u = min\{N_2, n - M_1\} = min\{9, n - 0\} = min\{9, n\} = \begin{cases} n \ for \ n \le 9 \\ 9 \ for \ n > 9 \end{cases}$$

$for\ 0 \le n \le 9$

$$y(n) = \sum_{k=0}^{n} x(k)\,h(n-k) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} = \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k} = \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left[\left(\frac{1}{2}\right)^{-1}\right]^{k} = \left(\frac{1}{2}\right)^{n} \left[\frac{1-\left[\left(\frac{1}{2}\right)^{-1}\right]^{n+1}}{1-\left(\frac{1}{2}\right)^{-1}}\right] = \left(\frac{1}{2}\right)^{n} \left[\frac{1-\left[\left(\frac{1}{2}\right)\right]^{n}}{-1}\right]$$

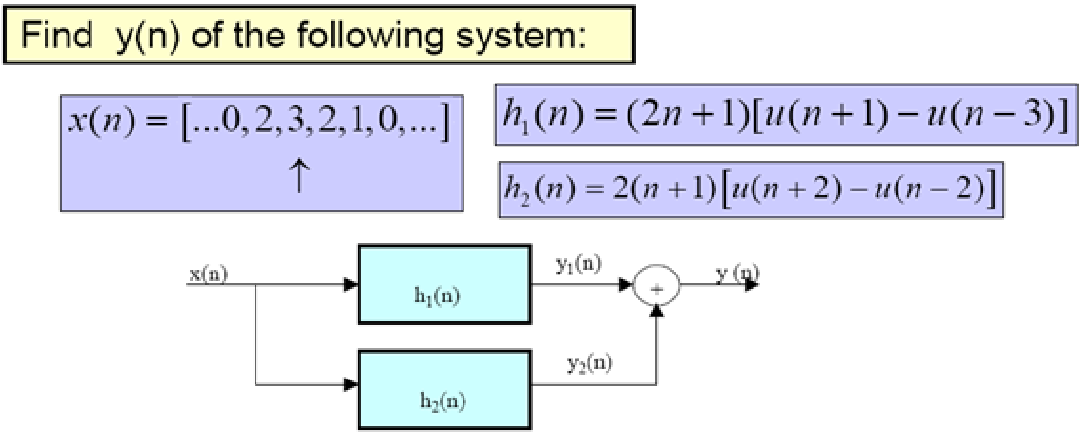
The last step above is obtained by using the geometrical progression formula given in appendix B (page 317 in (fundamentals of digital signal processing book) by

$$\sum_{k=0}^{n} (b)^{k} = \left[\frac{1-(b)^{n+1}}{1-b}\right] \qquad for\ b \ne 1$$

$for\ n > 9$

$$y(n) = \sum_{k=0}^{9} x(k)\,h(n-k) = \sum_{k=0}^{9} \left(\frac{1}{2}\right)^{n-k} = \left(\frac{1}{2}\right)^{n} \left[1 + \left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} + \cdots + \left(\frac{1}{2}\right)^{-9}\right]$$

H.W



De-convolution

1-Iterative method

Example: $h(n) = [4 \;\; 2 \;\; 3 \;\; -4\,]$, $y(n) = [\;4 \;\; 14 \;\; 17 \;\; 9 \;\; -6 \;\; -8\;]$

$find\; x(n)?$

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$$y(0) = \sum_{k=0}^{0} x(k)\, h(-k) = x(0) \cdot h(0)$$

$$x(0) = \frac{y(0)}{h(0)} = \frac{4}{4} = 1$$

$$y(1) = \sum_{k=0}^{1} x(k)\, h(1-k) = x(0) \cdot h(1) + x(1) \cdot h(0)$$

$$x(1) = \frac{y(1) - x(0)\cdot h(1)}{h(0)} = \frac{14-(1\times 2)}{4} = 3$$

$$y(2) = \sum_{k=0}^{2} x(k)\, h(2-k) = x(0) \cdot h(2) + x(1) \cdot h(1) + x(2) \cdot h(0)$$

$$x(2) = \frac{y(2) - x(0)\cdot h(2) - x(1)\cdot h(1)}{h(0)} = \frac{17-(1\times 3)-(3\times 2)}{4} = 2$$

$$y(3) = \sum_{\mathrm{k}=0}^{3} x(k)\, h(3-k) = x(0) \cdot h(3) + x(1) \cdot h(2) + x(2) \cdot h(1) + x(3) \cdot h(0)$$

$$x(3) = \frac{y(3) - x(0) \cdot h(3) - x(1) \cdot h(2) - x(2) \cdot h(1)}{h(0)} = 0$$

For $y(n) = x(n) * h(n)$, if length of $x(n) = N$, and length of $h(n) = M$ then length of $y(n) = (N+M) - 1$

For the above example, $N = 3$, and $M = 4$, then $(N+M) - 1 = 6$,

2-The Polynomial Method

Example: $h(n) = [4 \;\; 2 \;\; 3 \;\; -4\,] \Rightarrow \quad B(p) = 4 + 2p + 3p^2 - 4p^3$

$y(n) = [\;4 \;\; 14 \;\; 17 \;\; 9 \;\; -6 \;\; -8\;] \Rightarrow C(p) = 4 + 14p + 17p^2 + 9p^3 - 6p^4 - 8p^5$

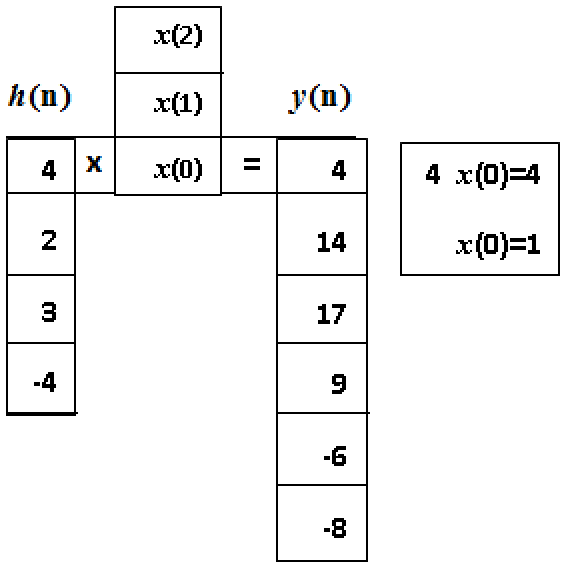
Since $y(n) = x(n) * h(n)$, then $C(p) = A(p) \cdot B(p)$ or $A(p) = \frac{C(p)}{B(p)}$

$$4 + 2p + 3p^2 - 4p^3 \;\Big)\; 4 + 14p + 17p^2 + 9p^3 - 6p^4 - 8p^5$$

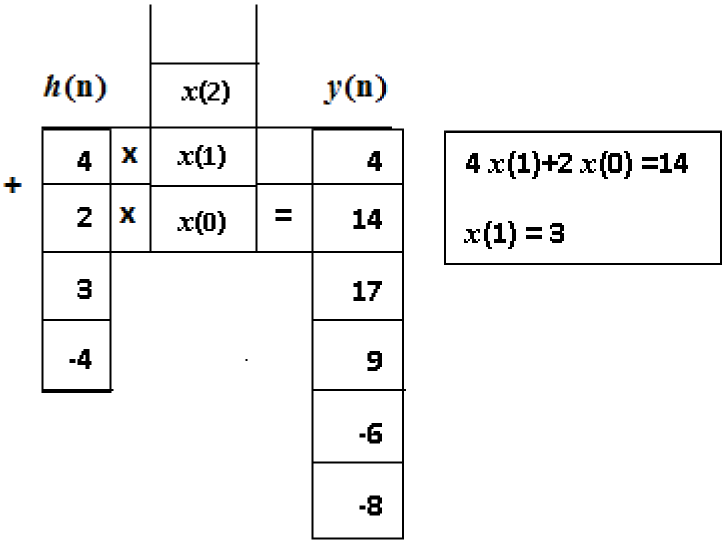
3-The Graphical Method

For the same example: $h(n) = [4 \quad 2 \quad 3 \quad -4\,]$ and $y(n) = [\;4 \quad 14 \quad 17 \quad 9 \quad -6 \quad -8\;]$

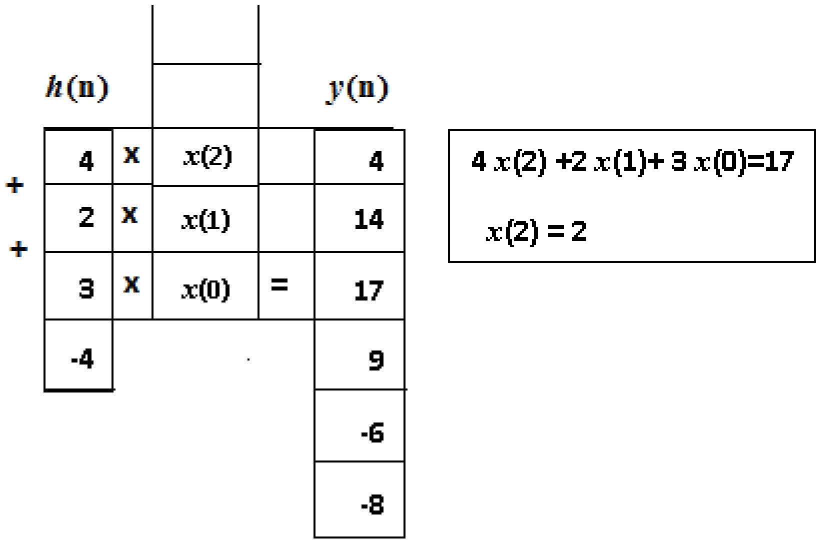
STEP ONE



STEP TWO



STEP THREE



If we calculate $x(3)$ and so on, they will be zero. Why?

Linear Constant-Coefficient Difference Equations (LCCDEs)

Remembering linear differential equations

$$\frac{dy(t)}{dt} - y(t) = x(t)$$

A difference equation is the discrete-time analogue of a differential equation. We simply use differences [x (n) - x (n-1)] rather than derivatives ($\frac{dx(t)}{dt}$).

An important subclass of linear systems are those whose input is $x(\mathrm{n})$, output is $y(\mathrm{n})$, and satisfying the following N^{th} - order LCCDE:

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r) \qquad a_0 \neq 0$$

If the system is causal, then we can rearrange the above Eq. as

$$y(n) = -\sum_{k=1}^{N} \frac{a_k}{a_0} y(n-k) + \sum_{k=0}^{M} \frac{b_r}{a_0} x(n-r)$$

Solutions of Linear Constant- Coefficient Difference Equations

First -order LCCDE

Example -1: Solve the following DE for y (n), assuming $y\,(\mathrm{n}) = 0$ for all $\mathrm{n} < 0$ and $x\,(\mathrm{n}) = \delta\,(\mathrm{n})$.

$$y\,(\mathrm{n}) - \mathrm{a}y\,(\mathrm{n}-1) = x\,(\mathrm{n})$$

This corresponds to calculating the response of the system when excited by an impulse, assuming "zero initial conditions"

Solution: Rewrite $\quad y(n) = ay(n-1) + x(n)$

Evaluate: $\quad y(0) = ay(-1) + x(0) = 1$

$$y(1) = ay(0) + x(1) = a$$

$$y(2) = ay(1) + x(2) = a^2$$

For all n>0, It can be written that

$$y(n) = a^n$$

Since the response of the system for $\mathrm{n} < 0$ is defined to be zero, the unit sample response becomes $h(n) = a^n u(n)$

If $|a| < 1$, then the system is ……………………?

If $|a| > 1$, then the system is ……………………?

N^{th} -order LCCDE

Two methods
Direct method
Indirect Method (z-transform)

Direct solution Method:

The total solution is the sum of two parts
Part 1 homogeneous solution
Part 2 particular solution

The Homogeneous solution

Assuming that the input . Since , this gives us the ***zero-input response*** of the system

$$\sum_{k=0}^{N} a_k y(n-k) = 0$$

The solution is the form of an exponential

$$y_h(n) = \lambda^n$$

substitute this in the previous equation.

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \ldots + a_{N-1}\lambda^{n-(N-1)} + a_N\lambda^{n-N} = 0$$

$$\lambda^{n-N}\left(\lambda^N + a_1\lambda^{N-1} + a_2\lambda^{N-2} + \ldots + a_{N-1}\lambda + a_N\right) = 0$$

This is called characteristic polynomial of the system. It has N roots and denotes by $\lambda_1, \lambda_2, \ldots, \lambda_N$

The roots can be real or complex or some roots are identical.

Let assume that roots are real and not identical, the solution becomes

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n + \dots + C_N\lambda_N^n$$

The coefficients $C_i, i = 1,2,\dots,N$ are determined from the initial conditions.

If there are two identical roots, the solution becomes

$$y_h(n) = C_1\lambda_1^n + C_1 n\lambda_1^n + C_2\lambda_2^n + \dots + C_N\lambda_N^n$$

Example-2:

Find the zero-input response for the second-order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The homogeneous solution form $y_h(n) = \lambda^n$

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\lambda_1 = -1,\ \lambda_2 = 4$$

The homogenous solution is

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n = C_1(-1)^n + C_2 4^n$$

The Particular solution:

Causal system is the output is depends only on present and past input signal

Input Signal $x(n)$	Particular Solution $y_p(n)$
A (constant)	K
AM^n	KM^n
An^M	$K_0 n^M + K_1 n^{M-1} + \dots + K_M$
$A^n n^M$	$A^n \left(K_0 n^M + K_1 n^{M-1} + \dots + K_M \right)$
$\begin{Bmatrix} A\cos\omega_0 n \\ A\sin\omega_0 n \end{Bmatrix}$	$K_1 \cos\omega_0 n + K_2 \sin\omega_0 n$

Example: -3 Find the particular solution for for

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \qquad x(n) = 2^n,\ n \ge 0$$

The particular solution form

$$y_p(n) = K2^n u(n) \qquad y_p(n) = K2^n,\ n \ge 0$$

$$K2^n u(n) = \frac{5}{6} K2^{n-1} u(n-1) - \frac{1}{6} K2^{n-2} u(n-2) + 2^n u(n)$$

Evaluate the equation for $n \ge 2$

$$4K = \frac{5}{6} 2K - \frac{1}{6} K + 4 \Rightarrow K = \frac{8}{5}$$

Therefore, the particular solution is

$$y_p(n) = \frac{8}{5} 2^n,\ n \ge 0$$

The total solution of the difference equation

$$y(n) = y_h(n) + y_p(n)$$

Example:-4

Determine the response $y(n),\ n \geq 0$ of the system described by the second-order difference equation

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$$

to the input $x(n) = 4^n u(n)$

The homogenous solution is

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 0$$

$$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$$

$$\lambda^{n+2}\left(\lambda^2 - 0.7\lambda + 0.1\right) = 0$$ → $\lambda_1 = 0.5$ → and $\lambda_2 = 0.2$

$$y_h(n) = c_1 0.5^n + c_2 0.2^n$$

Particular Solution:

$$y_p(n) = K4^n u(n)$$

$$K4^n u(n) - 0.7K4^{n-1}u(n-1) + 0.1K4^{n-2}u(n-2) = (2)4^n u(n) - 4^{n-2}u(n-2)$$

n=2

$$K4^2 - 0.7K4^1 + 0.1K4^0 = 2(4)^2 - 4^0$$

$$16K - 2.8K + 0.1K = 32 - 1$$ ⇨ $$K = \frac{31}{13.3} = 2.33$$

$$y_p(n) = 2.33(4)^n u(n)$$

The total solution

$$y(n) = \left[c_1 0.5^n + c_2 0.2^n + 2.33(4)^n\right]u(n)$$

To find c_1 and c_2

For n=0:

From difference equation,

$$y(0) = 0.7y(0-1) - 0.1y(0-2) + 2x(0) - x(0-2)$$
$$y(0) = 2$$

From the total solution,

$$y(0) = c_1 + c_2 + 2.33$$

For n=1:

From difference equation,

$$y(1) = 0.7y(1-1) - 0.1y(1-2) + 2x(1) - x(1-2)$$
$$y(1) = 1.4 + 8 = 9.4$$

From difference equation,

$$y(1) = 0.5c_1 + 0.2c_2 + 9.32$$

Therefore,

$$2 = c_1 + c_2 + 2.33 \qquad 9.4 = 0.5c_1 + 0.2c_2 + 9.32$$

$$c_2 = -0.807 \qquad c_1 = 0.466$$

Total Solution

$$y(n) = \left[0.466(0.5)^n - 0.807(0.2)^n + 2.33(4)^n\right]u(n)$$

Frequency Response of LTI Systems

The Fourier representation of signals plays an extremely important role in both continuous-time and discrete-time signal processing. It provides a method for mapping signals into another "domain" in which to manipulate them. What makes the Fourier representation particularly useful is the property that the convolution operation is mapped to multiplication. In addition, the Fourier transform provides a different way to interpret signals and systems. In this section, we will develop the discrete-time Fourier transform (*i.e.,* a Fourier transform for discrete-time signals). We will show how complex exponentials of linear time-invariant (LTI) systems and how this property leads to the notion of a frequency response representation of LSI systems.

A. Response to Complex Exponential

Let $x(n) = e^{j\omega n}$ be input into an LTI system with causal impulse response *h*(n). The output is

$$y(n) = h(n) * x(n) = h(n) * e^{j\omega n} = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, e^{j\omega(n-k)} = e^{j\omega n} \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k}$$

Let us define $H(e^{j\omega})$: a function of ω, as ω varies form $(-\infty\ to + \infty)$ to be

$$H\left(e^{j\omega}\right) \triangleq \sum_{k=-\infty}^{\infty} h(k)\, e^{-j\omega k}$$

$$y(n) = e^{j\omega n^{INPUT}} H\left(e^{j\omega}\right)^{frequency\ response}$$

or $$y(n) = x(n)\, H\left(e^{j\omega}\right)$$

- $H\left(e^{j\omega}\right)$ = frequency response function (a conjugate symmetric function of ω)

$$H\left(e^{j\omega}\right) = H_{Re}\left(e^{j\omega}\right) + jH_{Im}\left(e^{j\omega}\right) = \left|H\left(e^{j\omega}\right)\right| e^{\,j \arg H\left(e^{j\omega}\right)}$$
$$= \left|H\left(e^{j\omega}\right)\right| e^{\,j\,\emptyset\left(e^{j\omega}\right)}$$

- $\left|H\left(e^{j\omega}\right)\right|$= Magnitude response (an even function of ω)
- $\arg H\left(e^{j\omega}\right) = \emptyset\left(e^{j\omega}\right)$ = Phase response (an odd function of ω)
- $H\left(e^{j\omega}\right)$ is periodic with period = 2π

where the magnitude and phase of $H\left(e^{j\omega}\right)$ are given by

$$\left|H\left(e^{j\omega}\right)\right| = \left[{H_{Re}}^2\left(e^{j\omega}\right) + {H_{Im}}^2\left(e^{j\omega}\right)\right]^{1/2}$$

$$\arg H\left(e^{j\omega}\right) = \emptyset\left(e^{j\omega}\right) = \tan^{-1}\left[H_{Im}\left(e^{j\omega}\right)/H_{Re}\left(e^{j\omega}\right)\right]$$

B. Response to Sinusoidal

Now let $x(n) = A\ \cos(\omega_o n + \theta) = \frac{Ae^{j\omega_o n}\ e^{j\theta}}{2} + \frac{Ae^{-j\omega_o n}\ e^{-j\theta}}{2}$ be input into an LTI system with causal impulse response *h*(n).

Because of linearity the response can be found by adding the responses of the complex exponential sequences of $\frac{Ae^{j\omega_o n}\ e^{j\theta}}{2}$ and $\frac{Ae^{-j\omega_o n}\ e^{-j\theta}}{2}$ the output becomes

$$y(n) = A\,H\left(e^{j\omega_o n}\right) e^{j\omega_o n} e^{j\theta}\ /2 + A\,H\left(e^{-j\omega_o n}\right) e^{-j\omega_o n} e^{-j\theta}\ /2$$

The second part of *y*(n) is seen to be the complex conjugate of the first part, thus *y*(n) becomes two times the real part of either; that is

$$y(n) = 2\ Re\left[\frac{A}{2}\ H\left(e^{j\omega_o n}\right) e^{j\omega_o n}\ e^{j\theta}\ \right]$$

$$y(n) = 2\ Re\left[\frac{A}{2}\ \left|H\left(e^{j\omega_o}\right)\right|\ e^{\ j\,\emptyset\left(e^{j\omega_o}\right)}\ e^{j\omega_o n}\ e^{j\theta}\ \right]$$

$$y(n) = A\ Re\ \left\{\left|H\left(e^{j\omega_o}\right)\right|\ e^{[\ j\left(\omega_o n + \theta + \emptyset\left(e^{j\omega_o}\right)\right)]}\right\}$$

$$y(n) = A\ \left|H\left(e^{j\omega_o}\right)\right| \cos\left(\omega_o n + \theta + \emptyset\left(e^{j\omega_o}\right)\right)$$

Change in magnitude — Change in phase

Therefore, it has been shown that the output to sinusoid is another sinusoid of the same frequency but with different phase and different magnitude.

Example -1:- Find the frequency response of LTI system characterized by unit sample response (impulse response)

$$h(n) = a^n u(n) \quad for\ \ |a|\ < 1$$

<u>Solution:</u> It is an IIR system

By definition the frequency response $H\left(e^{j\omega}\right)$ is given by

$$H\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} h(n)\ e^{-j\omega n} = \sum_{n=0}^{\infty} a^n\ e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty}\left(a\ e^{-j\omega}\right)^n = \frac{1}{1-a\ e^{-j\omega}} = \frac{1}{(1-a\cos\omega)+j(a\sin\omega)}$$

Mag. response= $\left|H\left(e^{j\omega}\right)\right| = \frac{1}{[(1-a\cos\omega)^2 + \ (a\sin\omega)^2]^{1/2}} = \frac{1}{(1+a^2-2\,a\cos\omega)^{1/2}}$

Phase response= $\emptyset\left(e^{j\omega}\right) = -\tan^{-1}[a\sin\omega/(1-a\cos\omega)]$

<u>Plot Mag. and Phase responses for $a = 0.5$.</u>

Example -2:- For an LTI discrete-time system with the following impulse response

$$h(n) = \delta(n-1) - 2\,\delta(n-3) + \delta(n-5)$$

(i) Give expressions for $h(n)$ in terms of unit steps and then in vector form.

(ii) Plot such impulse response $h(n)$.

(iii) Specify whether the system is of the FIR or of the IIR type, Why?

(iv) Find the frequency response $H(e^{j\omega})$.

(v) Find and plot magnitude and phase responses.

(vi) Compute the unit step response.

(vii) Compute the response to $x(n) = 4\cos[\frac{\pi}{4}(n-2)]$. Then calculate the corresponding time delay of the system if the sampling rate is 8 k sample/sec.

Solution:

(i) Using the fact that a unit sample can be written as the difference of two steps as follows:

$$\delta(n) = u(n) - u(n-1)$$

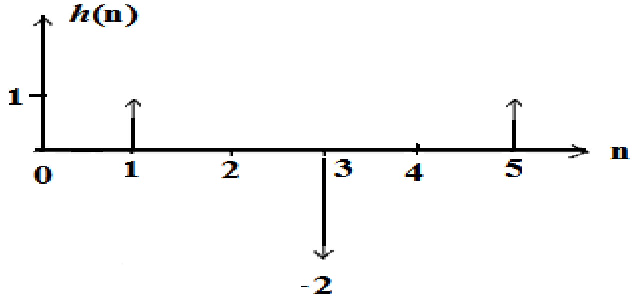
Therefore, $h(n) = [\,u(n-1) - u(n-2)] - 2[\,u(n-3) - u(n-4)] + [\,u(n-5) - u(n-6)]$

i.e., $h(n) = u(n-1) - u(n-2) - 2\,u(n-3) + 2\,u(n-4) + u(n-5) - u(n-6)$

In vector form,

$h(n) = [\,0 \quad 1 \quad 0 \quad -2 \quad 0 \quad 1]$

(ii) The plot of such impulse response $h(n)$ is shown here.



(iii) The system is of the FIR type, because $h(n)$ is of finite duration.

(iv) $H(e^{j\omega}) = \sum_{-\infty}^{\infty} h(n)\, e^{-j\omega n} = e^{-j\omega} - 2\,e^{-j3\omega} + e^{-j5\omega} =$

$$= e^{-j3\omega}\,[\,e^{j2\omega} - 2 + e^{-j2\omega}] = e^{-j3\omega}[-2 + 2\cos(2\omega)]$$

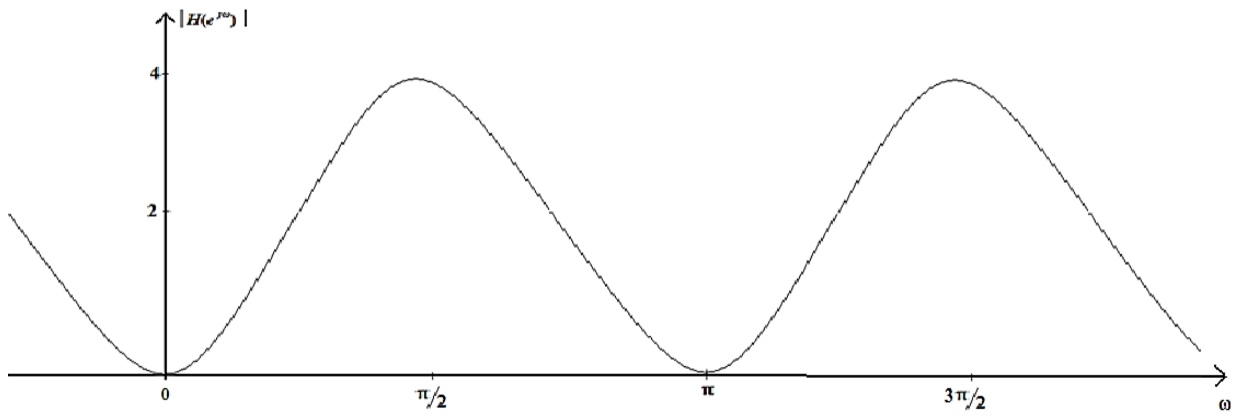
$$= e^{-j3\omega}\,(-1)\,[2 - 2\cos(2\omega)] = e^{-j3\omega}\,(e^{-j\pi})\,[2 - 2\cos(2\omega)]$$

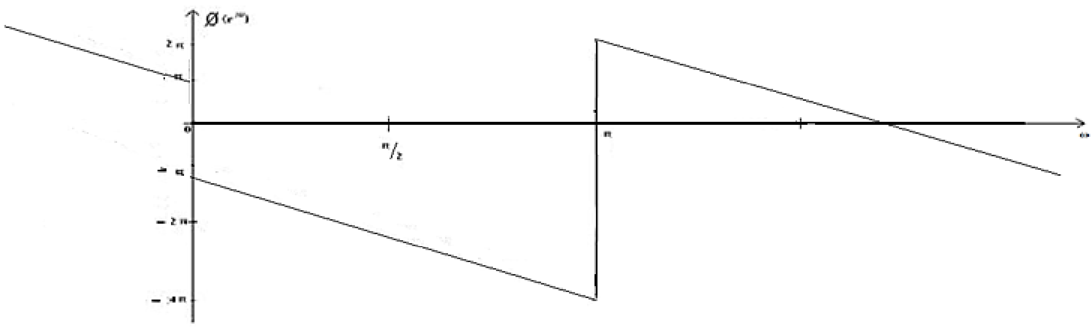
$$= e^{-j(3\omega+\pi)}\,[2 - 2\cos(2\omega)]$$

(v) From the above frequency response,

Magnitude response = $|H(e^{j\omega})| = [2 - 2\cos(2\omega)]$.

Phase response = $\emptyset(e^{j\omega}) = -(3\omega + \pi) = -3\omega - \pi$





(vi) The unit step response is $u(n) * h(n) = u(n) * [\ \delta(n-1) - 2\,\delta(n-3) + \delta(n-5)\ ]$

$= u(n-1) - 2\,u(n-3) + u(n-5)$.

(vii) The response to $x(n) = 4\cos[\frac{\pi}{4}(n-2)]$

$\omega_0 = \frac{\pi}{4}$, So $|H(e^{j\omega_0})| = [2 - 2\cos(2\omega_0)] = \left[2 - 2\cos\left(\frac{\pi}{2}\right)\right] = 2.$

$\emptyset(e^{j\omega_0}) = -3\,\omega_0 - \pi = -\frac{3\pi}{4} - \pi = -\frac{7\pi}{4}$

$y(n) = 4 \cdot (2) \cdot \cos[\frac{\pi}{4}\ n - \frac{\pi}{2} - \frac{7\pi}{4})] = 8 \cdot \cos[\frac{\pi}{4}\ (n - 2 - 7)]$

$= 8 \cdot \cos[\frac{\pi}{4}\ (n - 9)]$

Delay = 9−2=7 samples, Time delay = 7 T_s = 7/f_s = 7/8000 = 0.000875 sec.

= 0.875 m sec.

Example -3:- If the step response of an LTI system is

$y_u(n) = [\,1\ \ 3\ \ 2\ \ 2\ \ 3\ \ 1\,]$,

find the unit sample response h(n). Then find magnitude and phase responses. Is the system possesses a linear phase response? Plot it.

Solution:

It is known that $\delta(n) = u(n) - u(n-1)$, So

$$h(n) * \delta(n) = h(n) * [u(n) - u(n-1)]$$

or $$h(n) * \delta(n) = h(n) * u(n) - h(n) * u(n-1),$$

i.e., $$h(n) = y_u(n) - y_u(n-1) = [\,1\ \ 3\ \ 2\ \ 2\ \ 3\ \ 1\ \ 0\,] - [\,0\ \ 1\ \ 3\ \ 2\ \ 2\ \ 3\ \ 1\,]$$

$$h(n) = [1\ \ 2\ \ -1\ \ \ 0\ \ 1\ \ -2\ \ \ -1]$$

$$H(e^{j\omega}) = 1 + 2\,e^{-j\omega} - e^{-j2\omega} + 0\,.\,e^{-j3\omega} + e^{-j4\omega} - 2\,e^{-j5\omega} - e^{-j6\omega}$$

$$H(e^{j\omega}) = e^{-j3\omega}\,[\,(e^{j3\omega} - e^{-j3\omega}) + 2\,(e^{j2\omega} - e^{-j2\omega}) - (e^{j\omega} - e^{-j\omega})\,]$$

$$H(e^{j\omega}) = e^{-j3\omega}\,[\,2j\sin(3\omega) + 4\,j\sin(2\omega) - 2\,j\sin(\omega)\,]$$

$$H(e^{j\omega}) = je^{-j3\omega}\,[\,2\sin(3\omega) + 4\sin(2\omega) - 2\sin(\omega)\,]$$

$$H(e^{j\omega}) = e^{j\pi/2}\,e^{-j3\omega}\,[\,2\sin(3\omega) + 4\sin(2\omega) - 2\sin(\omega)\,]$$

$$H(e^{j\omega}) = e^{-j(3\omega - \frac{\pi}{2})}\,[\,2\sin(3\omega) + 4\sin(2\omega) - 2\sin(\omega)\,]$$

Mag. Response = $|H(e^{j\omega})| = 2\sin(3\omega) + 4\sin(2\omega) - 2\sin(\omega)$

Phase Response = $\emptyset(e^{j\omega}) = -3\omega + {}^{\pi}/_{2}$ *; Yes linear phase, plot it.*