

Lecture 6

Friction

Tangential forces generated between contacting surfaces are called friction forces and occur to some degree in the interaction between all real surfaces. Whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency. In some types of machines and processes we want to minimize the retarding effect of friction forces. Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere. In other situations we wish to maximize the effects of friction, as in brakes, clutches, belt drives, and wedges. Wheeled vehicles depend on friction for both starting and stopping, and ordinary walking depends on friction between the shoe and the ground.

Friction forces are present throughout nature and exist in all machines no matter how accurately constructed or carefully lubricated. A machine or process in which friction is small enough to be neglected is said to be ideal. When friction must be taken into account, the machine or process is termed real. In all real cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat. Wear is another effect of friction.

Friction Phenomena

Types of Friction

(a) Dry Friction. Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always opposes the motion or impending motion. This type of friction is also called Coulomb friction. The principles of dry or Coulomb friction were developed largely from the experiments of Coulomb in 1781 and from the work of Morin from 1831 to 1834. Although we do not yet have a comprehensive theory of dry friction, in Art. 6/3 we describe an analytical model sufficient to handle the vast majority of problems involving dry friction.

(b) Fluid Friction. Fluid friction occurs when adjacent layers of fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction. Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers. Fluid friction is treated in the study of fluid mechanics and will not be discussed further in this book.

(c) Internal Friction. Internal friction occurs in all solid materials which are subjected to cyclical loading. For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction. For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this

deformation. The mechanism of internal friction is associated with the action of shear deformation, which is discussed in references on materials science.

Dry Friction

Mechanism of Dry Friction

Consider a solid block of mass m resting on a horizontal surface, as shown in Fig. 1a. We assume that the contacting surfaces have some roughness. The experiment involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of P is shown in Fig. 1b, where the tangential friction force exerted by the plane on the block is labeled "F". This friction force acting on the body will always be in a direction to oppose motion or the tendency toward motion of the body. There is also a normal force N which in this case equals mg , and the total force R exerted by the supporting surface on the block is the resultant of N and F .

A magnified view of the irregularities of the mating surfaces, Fig. 1c, helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps. The direction of each of the reactions on the block, R_1 , R_2 , R_3 , etc. depends not only

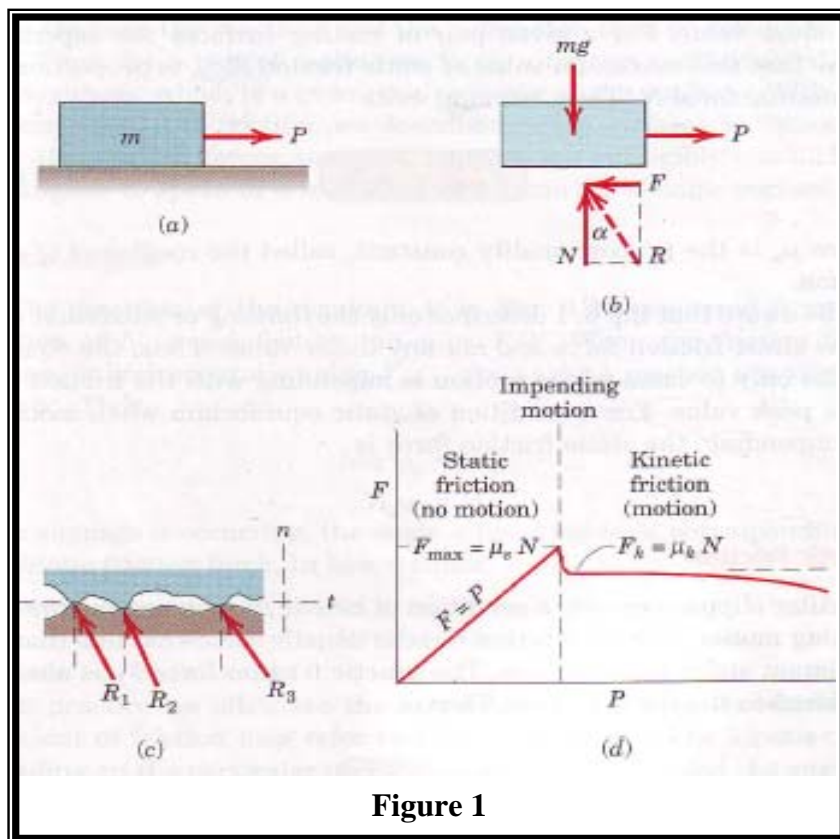


Figure 1

on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force N is the sum of the n-components of the R 's, and the total frictional force F is the sum of the t-components of the R 's. when the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R 's are smaller than when the surfaces are at rest relative to one another. This observation helps to explain the well known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.

If we perform the experiment and record the friction force F as a function of P , we obtain the relation shown in Fig. 1d. when P is zero, equilibrium requires that there be no friction force. As p is increased the friction force must be equal and opposite to p as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.

Static Friction

The region in Fig. 1d up to the point of slippage or impending motion is called the range of static friction, and in this range the value of the friction force is determined by the equations of equilibrium. This friction force may have any value from zero up to and including the maximum value. For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{\max} is proportional the normal force N . Thus. we may write

$$F_{\max} = \mu_s N \quad \dots \text{equ. 1}$$

where μ_s is the proportionality constant, called the coefficient of static friction. Be aware that Eq. 1 describes only the limiting or maximum value Of the static friction force and not any lesser value. Thus, the equation applies only to cases where motion is impending with the friction force at its peak value. For a condition of static equilibrium when motion is not impending, the static friction force is

$$F < \mu_s N$$

Kinetic Friction

After slippage occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k , is also proportional to the normal force. Thus.

$$F_k = \mu_k N \quad \dots \text{equ. 2}$$

where μ_k is the coefficient of kinetic friction. It follows that μ_k is generally less than μ_s . As the velocity of the block increases, the kinetic friction decreases somewhat, and at high velocities, this decrease may be significant. Coefficients of friction depend greatly on the exact condition of the surfaces, as well as on the relative velocity, and are subject to considerable uncertainty.

Because of the variability of the conditions governing the action friction, in engineering practice it is frequently difficult to distinguish between a static and a kinetic coefficient, especially in the region of transition between impending motion and motion. Well-greased screw threads under mild loads, for example, often exhibit comparable frictional resistance whether they are on the verge of turning or whether they are in motion.

In the engineering literature we frequently find expressions for maximum static friction and for kinetic friction written simply as $F = \mu N$. It is understood from the problem at hand whether maximum static friction or kinetic friction is described. Although we will frequently distinguish between the static and kinetic coefficients, in other cases no distinction will be made, and the friction coefficient will be written simply as μ . In those cases you must decide which of the friction conditions, maximum static friction for impending motion or kinetic friction, is involved. We emphasize again that many problems involve a static friction force which is less than the maximum value at impending motion, and therefore under these conditions the friction relation Eq. 1 cannot be used.

Figure 1c shows that rough surfaces are more likely to have larger angles between the reactions and the n-direction than do smoother surfaces. Thus, for a pair of mating surfaces, a friction coefficient reflects the roughness, which is a geometric property of the surfaces. With this geometric model of friction, we describe mating surfaces as "smooth" when the friction forces they can support are negligibly small. It is meaningless to speak of a coefficient of friction for a single surface.

Factors Affecting Friction

Further experiment shows that the friction force is essentially independent of the apparent or projected area of contact. The true contact area is much smaller than the projected value, since only the peaks of the contacting surface irregularities support the load. Even relatively small normal loads result in high stresses at these contact points. As the normal force increases, the true contact area also increases as the material undergoes yielding, crushing, or tearing at the points of contact.

A comprehensive theory of dry friction must go beyond the mechanical explanation presented here. For example, there is evidence that molecular attraction may be an important cause of friction under conditions where the mating surfaces are in very close contact. Other factors which influence dry friction are the generation of high local temperatures and adhesion at contact points, relative hardness of mating surfaces, and the presence of thin surface films of oxide, oil, dirt, or other substances,

Types of Friction Problems

We can now recognize the following three types of problems encountered in applications involving dry friction. The first step in solving a friction problem is to identify its type.

(1) In the first type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on the verge of slipping, and the friction force equals the limiting static friction $F_{\max} = \mu_s N$. The equations of equilibrium will, of course, also hold.

(2) In the second, type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force F necessary for equilibrium. Three outcomes are possible:

(a) $F < (F_{\max} = \mu_s N)$: Here the friction force necessary for equilibrium can be supported, and therefore the body is in static equilibrium as assumed. We emphasize that the actual friction force F is less than the limiting value F_{\max} given by Eq. 1 and that F is determined solely by the equations of equilibrium.

(b) $F = (F_{\max} = \mu_s N)$: Since the friction force F is at its maximum value F_{\max} motion impends, as discussed in problem type (1). The assumption of static equilibrium is valid.

(c) $F > (F_{\max} = \mu_s N)$: Clearly this condition is impossible, because the surfaces cannot support more force than the maximum $\mu_s N$. The assumption of equilibrium is therefore invalid, and motion occurs. The friction force F is equal to $\mu_s N$ from Eq. 2.

(3) In the third type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies. For this problem type, Eq.2 always gives the kinetic friction force directly.

The foregoing discussion applies to all dry contacting surfaces and to a limited extent, to moving surfaces which are partially lubricated.

Examples

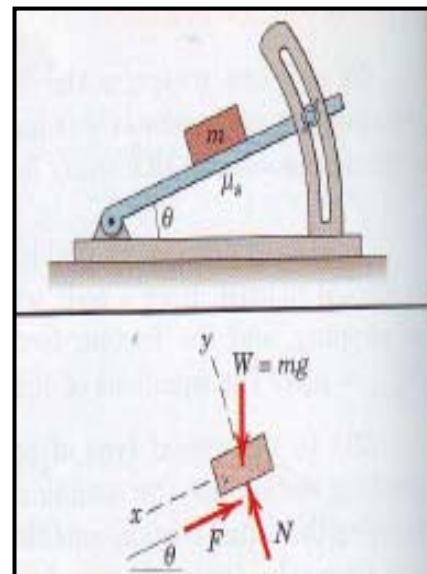
Example 1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s .

Solution

The free-body diagram of the block shows its weight $W = mg$, the normal force N , and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present. Equilibrium in the x - and y -directions requires

$$\begin{aligned} [\Sigma F_x = 0] \quad \quad \quad mg \sin \theta - F &= 0 & F &= mg \sin \theta \\ [\Sigma F_y = 0] \quad \quad \quad -mg \cos \theta + N &= 0 & N &= mg \cos \theta \end{aligned}$$



Dividing the first equation by the second gives $F/N = \tan \theta$. Since the maximum angle occurs when $F = F_{\max} = \mu_s N$, for impending motion we have

$$\mu_s = \tan \theta_{\max} \quad \text{or} \quad \theta_{\max} = \tan^{-1} \mu_s \quad \text{Ans.}$$

Example 2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.

Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

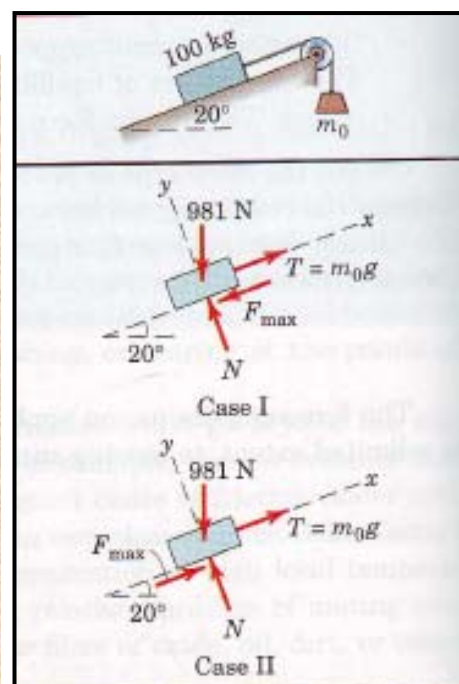
$$\begin{aligned} [\Sigma F_y = 0] \quad \quad \quad N - 981 \cos 20^\circ &= 0 & N &= 922 \text{ N} \\ [F_{\max} = \mu_s N] \quad \quad \quad F_{\max} &= 0.30(922) = 277 \text{ N} \\ [\Sigma F_x = 0] \quad \quad \quad m_0(9.81) - 277 - 981 \sin 20^\circ &= 0 & m_0 &= 62.4 \text{ kg} \quad \text{Ans.} \end{aligned}$$

The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x -direction requires

$$[\Sigma F_x = 0] \quad \quad \quad m_0(9.81) + 277 - 981 \sin 20^\circ = 0 \quad m_0 = 6.01 \text{ kg} \quad \text{Ans.}$$

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{\max} and N be concurrent with the 981-N weight and the tension T .



Example 3

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surface is shown. Determine the maximum value which P may have before any slipping takes place.

Solution. The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

The normal forces, which are in the y -direction, may be determined without reference to the friction forces, which are all in the x -direction. Thus,

$$[\Sigma F_y = 0] \quad (30\text{-kg}) \quad N_1 - 30(9.81) \cos 30^\circ = 0 \quad N_1 = 255 \text{ N}$$

$$(50\text{-kg}) \quad N_2 - 50(9.81) \cos 30^\circ - 255 = 0 \quad N_2 = 680 \text{ N}$$

$$(40\text{-kg}) \quad N_3 - 40(9.81) \cos 30^\circ - 680 = 0 \quad N_3 = 1019 \text{ N}$$

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

$$[F_{\max} = \mu_s N] \quad F_1 = 0.30(255) = 76.5 \text{ N} \quad F_2 = 0.40(680) = 272 \text{ N}$$

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$$[\Sigma F_x = 0] \quad P - 76.5 - 272 + 50(9.81) \sin 30^\circ = 0 \quad P = 103.1 \text{ N}$$

We now check on the validity of our initial assumption. For the 40-kg block with $F_2 = 272 \text{ N}$ the friction force F_3 would be given by

$$[\Sigma F_x = 0] \quad 272 + 40(9.81) \sin 30^\circ - F_3 = 0 \quad F_3 = 468 \text{ N}$$

But the maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459 \text{ N}$. Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F_3 = 459 \text{ N}$, equilibrium of the 40-kg block for its impending motion requires

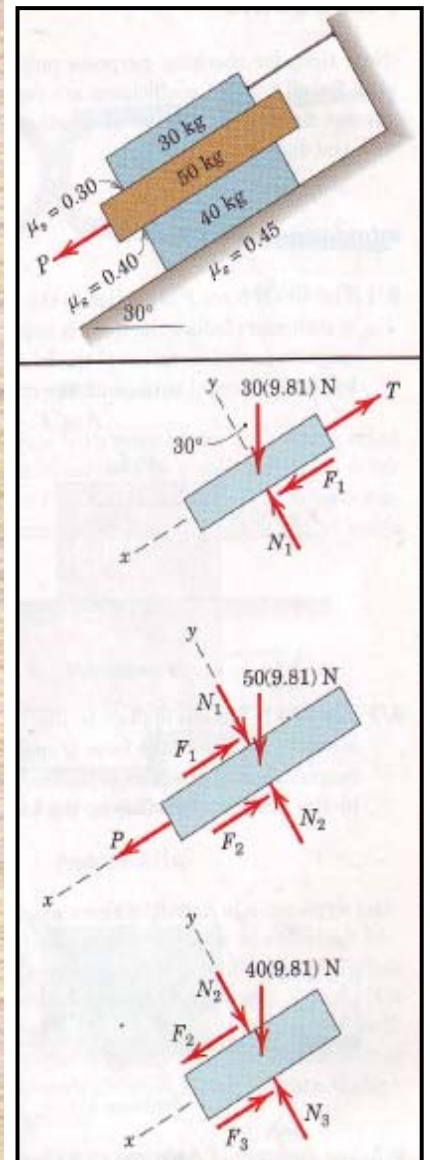
$$[\Sigma F_x = 0] \quad F_2 + 40(9.81) \sin 30^\circ - 459 = 0 \quad F_2 = 263 \text{ N}$$

Equilibrium of the 50-kg block gives, finally,

$$[\Sigma F_x = 0] \quad P + 50(9.81) \sin 30^\circ - 263 - 76.5 = 0$$

$$P = 93.8 \text{ N} \quad \text{Ans.}$$

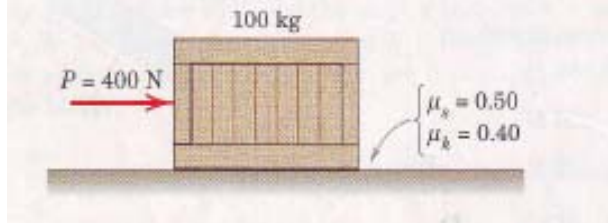
Thus, with $P = 93.8 \text{ N}$, motion impends for the 50-kg and 40-kg blocks as a unit.



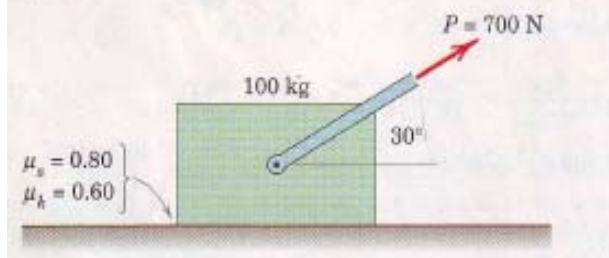
Problems

The 400-N force P is applied to the 100-kg crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate.

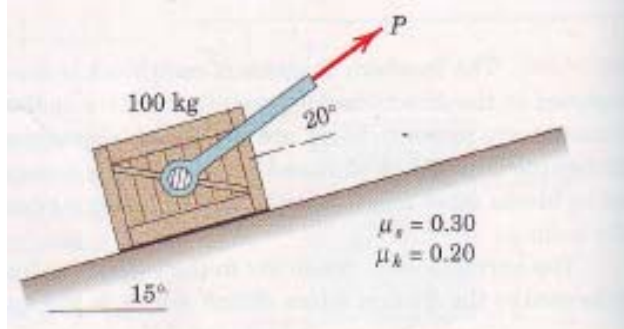
Ans. $F = 400$ N to the left



The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.

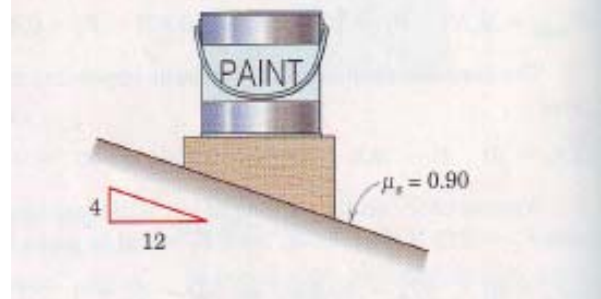


The coefficients of static and kinetic friction between the 100-kg block and the inclined plane are 0.30 and 0.20, respectively. Determine (a) the friction force F acting on the block when P is applied with a magnitude of 200 N to the block at rest, (b) the force P required to initiate motion up the incline from rest, and (c) the friction force F acting on the block if $P = 600$ N.

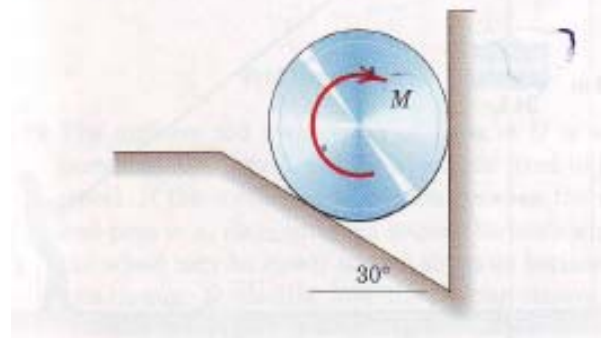


The 1.2-kg wooden block is used for level support of the 9-kg can of paint. Determine the magnitude and direction of (a) the friction force exerted by the roof surface on the wooden block and (b) the total force exerted by the roof surface on the wooden block.

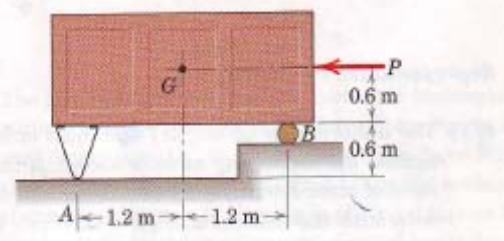
Ans. (a) $F = 31.6$ N, (b) $P = 100.1$ N up



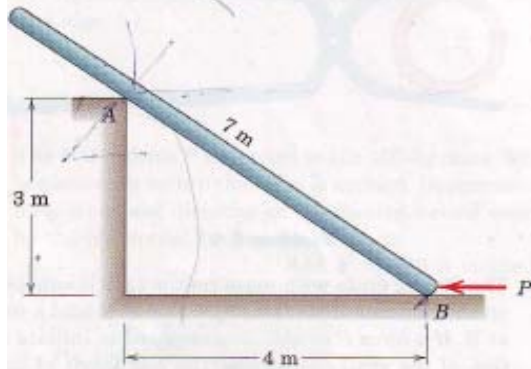
The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surfaces is 0.30, calculate the applied clockwise couple M which would cause the cylinder to slip.



The 200-kg crate with mass center at G is supported on the horizontal surfaces by a skid at A and a roller at B . If a force P of 400 N is required to initiate motion of the crate, determine the coefficient of static friction at A .

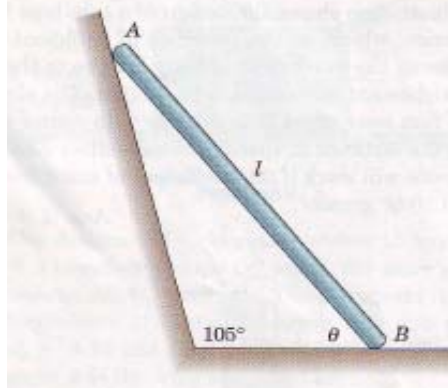


2 The uniform 7-m pole has a mass of 100 kg and is supported as shown. Calculate the force P required to move the pole if the coefficient of static friction for each contact location is 0.40.

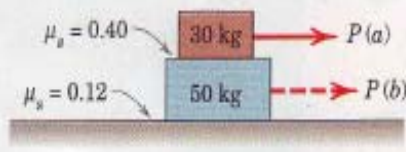


The uniform pole of length l and mass m is placed against the supporting surfaces shown. If the coefficient of static friction is $\mu_s = 0.25$ at both A and B, determine the maximum angle θ at which the pole can be placed before it begins to slip.

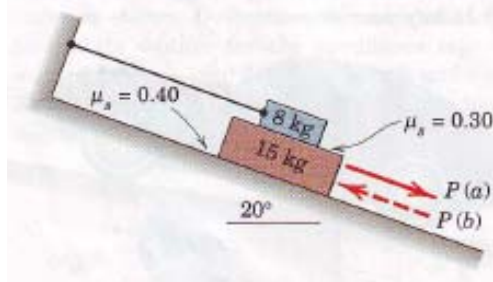
Ans. $\theta = 59.9^\circ$



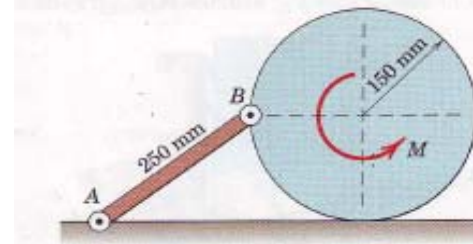
The force P is applied to (a) the 30-kg block and (b) the 50-kg block. For each case, determine the magnitude of P required to initiate motion.



The two blocks are placed on the incline with the cable taut. (a) Determine the force P required to initiate motion of the 15-kg block if P is applied down the incline. (b) If P is applied up the incline and slowly increased from zero, determine the value of P which will cause motion and describe that motion.

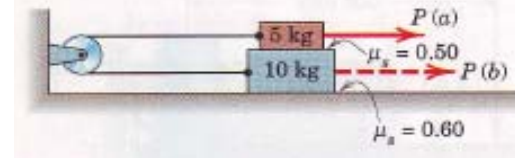


The strut AB of negligible mass is hinged to the horizontal surface at A and to the uniform 25-kg wheel at B. Determine the minimum couple M applied to the wheel which will cause it to slip if the coefficient of static friction between the wheel and the surface is 0.40.

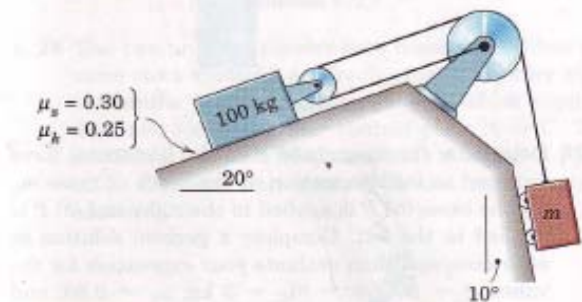


The system of two blocks, cable, and fixed pulley is initially at rest. Determine the horizontal force P necessary to cause motion when (a) P is applied to the 5-kg block and (b) P is applied to the 10-kg block. Determine the corresponding tension T in the cable for each case.

Ans. (a) $P = 137.3 \text{ N}$, $T = 112.8 \text{ N}$
(b) $P = 137.3 \text{ N}$, $T = 24.5 \text{ N}$



Determine the range of mass m for which the 100-kg block is in equilibrium. All wheels and pulleys have negligible friction.



Determine the magnitude P of the horizontal force required to initiate motion of the block of mass m_0 for the cases (a) P is applied to the right and (b) P is applied to the left. Complete a general solution in each case, and then evaluate your expression for the values $\theta = 30^\circ$, $m = m_0 = 3 \text{ kg}$, $\mu_s = 0.60$, and $\mu_k = 0.50$.

