- One of the most important things to know about a variable is its distribution. Knowledge of the probability distribution of the variables provides the clinicians and researchers with a powerful tool for summarization and describing a set of data and for reaching conclusion about population on the basis of a sample drawn from that population. We have several types of distribution in statistics, but the "**normal distribution**" is the most important one.
- A probability distribution defines the relationship between the outcomes and their likelihood of occurrence. We have several types of distribution in statistics:

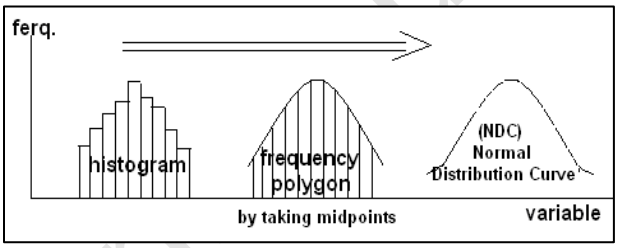
I. For discrete variables we have

a) Binomial Distribution: dichotomous outcomes (A-B, heads-tails, yes-no, on-off, is-is not, right-wrong, etc.)
b) Poisson Distribution Useful for studying rare random events.

II. But the "Normal distribution" "Gaussian distribution", for continues variables is the most important one. This is because:

- Many human variables naturally have a "bell shaped" distribution.
- The distributions are tied to probabilities, and it is the probability which will be of interest to us

- If we have a group of continuous variables with certain class interval, we can represent them by histogram and frequency polygon. But suppose we have a group of variables which is huge and the class interval is very small so the frequency polygon will take a shape of very smooth curve & that curve is called "normal distribution curve"

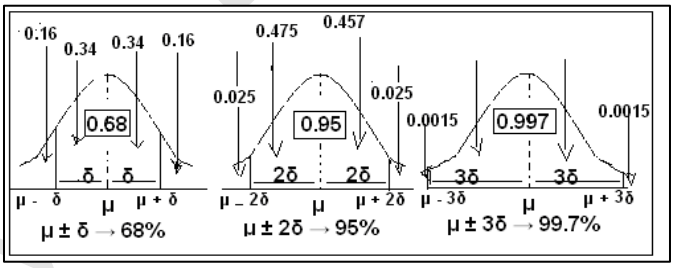


The normal distribution "Gaussian distribution", "Bell Shaped distribution" is the most important distribution in the statistics, the parameters of this distributions are:

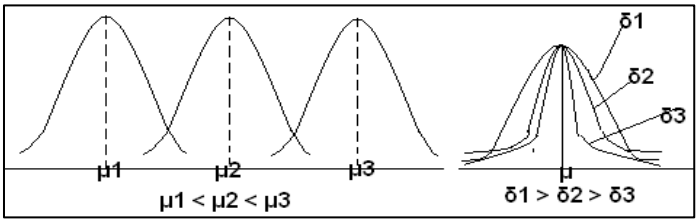
1) The mean (μ) → Measure of location.
2) The standard deviation (∂) →Measure of dispersion.

Characteristic of the normal distribution

1) Used for the continuous variables, between
2) Symmetrical about its mean (μ), ((either side of mean is a mirror image of other side.
3) Mean, median, and mode are equal.
4) The total area under the curve is equal to one, 50% on the left &50% on the right of a perpendicular erected at the mean.
5) The normal distribution is completely determined by the parameters (μ) & (∂). Different values of μ shift the graph along the X-axis, while different values of ∂ shift the graph along the Y-axis (determine the degree of flatness or peakness of the graph).
6) μ ± 1∂ → 68% of the area.
μ ± 2∂ → 95% of the area.
μ ± 3∂ → 99.7% of the area.

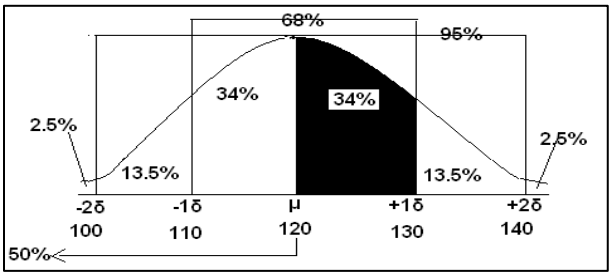


Different values of μ and δ shift the graph of distribution along X & Y axes. If we change μ while keeping δ constant, the curve will shift to the right on increasing μ & to the left on decreasing μ. On changing δ and keeping μ constant; the curve will become more flat on increasing δ and narrower on decreasing δ without any shifting the curve to any side. μ1 < μ2 < μ3



- Since we know the shape of the curve, we can (using calculus) calculate the area under the curve
- The percentage of that area can be used to determine the probability that a given value could be pulled from a given distribution.
- Each normal distribution with its own values of m and s (unit) would need its own calculation of the area under various points on the curve

Ex: If population mean of systolic blood pressure is 120 mmHg with population standard deviation of 10 mmHg. What is the probability of getting a patient with systolic BP a) between 120 and 130 mmHg, b) < 120mmHg, c) < 100 mmHg d) between 120 and 125 mmHg?



Answers:

a) From 120 to 130 we move one δ, so the probability is 34% (0.34) (i.e. half of 68%).

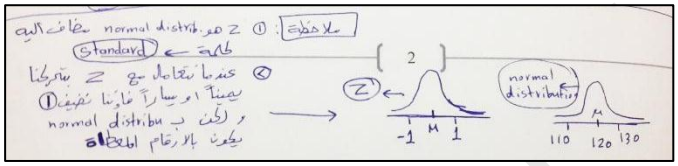
b) Probability of less than 120 mmHg is 50%.

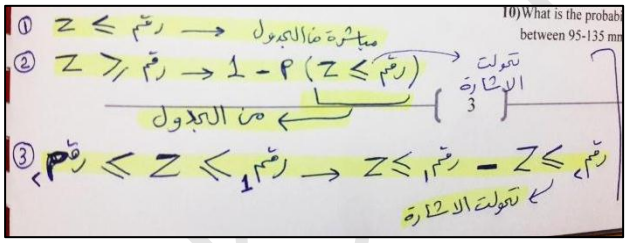
c) Probability of less than 100 mmHg is 2.5%.

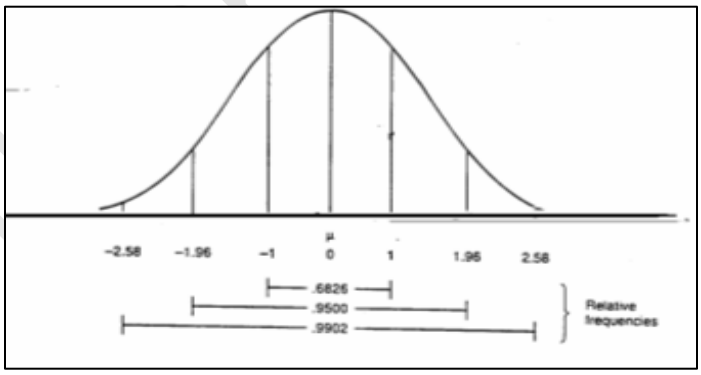
d) Probability of SBP between 120 and 125 mmHg; we must follow Z scale.

The standard normal distribution "Z-distribution".

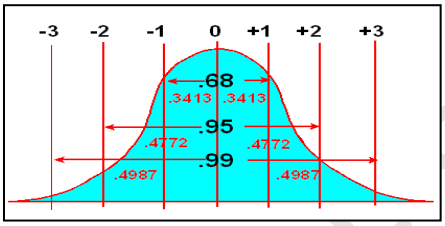
It's the normal distribution curve which has a mean of zero and a standard deviation of one **(μ=0, & ∂ =1).** → **Z = x - μ/ ∂**

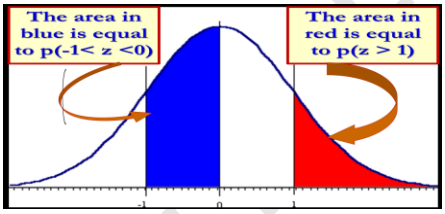






If we know the population means and population standard deviation, for any value of X we can compute a z-score by subtracting the population mean and dividing the result by the population standard deviation

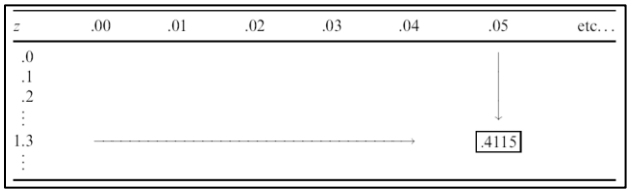




Properties of Z Distribution (Z-score)

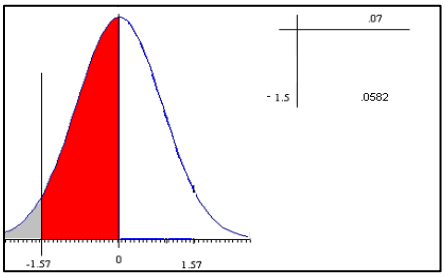
- 90% of the values of a normal variable lie within ± 1.65 sample standard deviations from the sample mean
- 95% of the values of a normal variable lie within ± 1.96 sample standard deviations from the sample mean
- 99% of the values of a normal variable lie within ± 2.58 sample standard deviations from the sample mean

How to Read Z Table ((Must understand Z table, area to the left))



Ex: Find p(Z<-1.57)

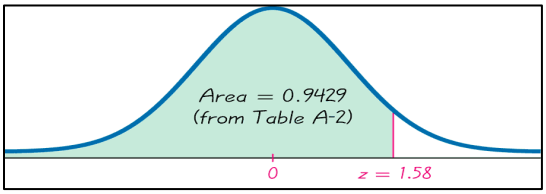
From Z table=0.0582



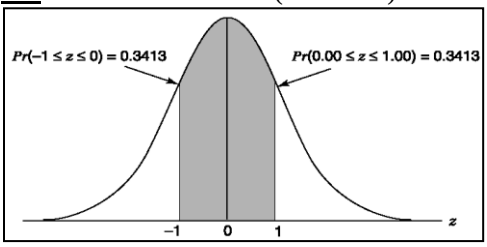
EX: From Z table: Find P ($z \geq 1.58$)

P($z < 1.58$) = 0.9429

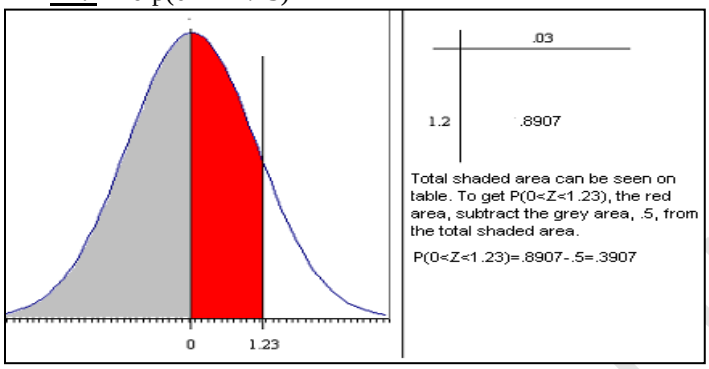
P($z > 1.58$) = 1 - 0.9429 = 0.0571



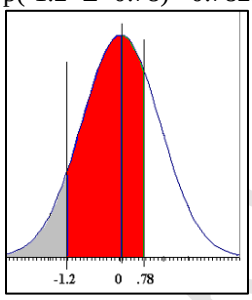
Ex: From Z table find: Pr(-1 < z < 1) = 0:6826



Ex: Find p(0<Z<1.23)



Ex: Calculate p(-1.2<Z<0.78)
p(-1.2<Z<0.78)= 0.7823-0.1151=0.6672



Ex: Calculate p(-1.2<Z<0.78)
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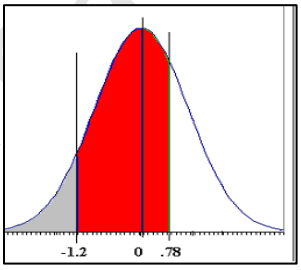

Ex: What is the probability of having a patient with B.P between 110-130 mm Hg? μ=120, ∂ =10. ((Suppose the B.P is normally distributed)).

Z = x - μ/ ∂
= 110-120/10 =-1
= 130-120/10 = +1

P (110≤ x ≤ 130) → P(-1≤ Z ≤+ 1). &From the Z-table, P=0.68.

Ex2: What is the probability of having a patient with B.P above 140mm Hg?

Z = x - μ/ ∂
= 140-120/10 = +2

P(x ≥ 140) → P (Z ≥ +2). &From the Z-table, P=0.023.

Ex: If the total cholesterol values for a certain target population are approximately normally distributed with a mean of 200 (mg/100 mL) and a standard deviation of 20 (mg/100 mL), what is the probability that a person picked at random from this population will have a cholesterol value greater than 240 (mg/100 mL)?

Z = x - μ/ ∂ = 240-200/ 20 = 2
P (x >240) → P(Z > 2). = 0:0228 or 2.28%

Ex: in certain population the mean of SBP (μ=120), and ∂ =10mmHg. What is the probability of having a patient with B.P between 110-130 mm Hg?

1) What is the probability of having a patient with B.P between 105-125 mm Hg?
2) What is the probability of having a patient with B.P ≤ 100 Hg?
3) What is the probability of having a patient with B.P ≥ 135 mm Hg?
4) What is the probability of having a patient with B.P between 120-140 mm Hg?
5) What is the probability of having a patient with B.P between 100-140 mm Hg?
6) What is the probability of having a patient with B.P between 90-150 mm Hg?
7) What is the probability of having a patient with B.P between ≥ 150 mm Hg?
8) What is the probability of having a patient with B.P between ≤150 mm Hg?
9) What is the probability of having a patient with B.P between 140-150 mm Hg?

10) What is the probability of having a patient with B.P between 95-135 mm Hg?

EX: IQ's are normally distributed with mean 100 and standard deviation 15. Find the probability that a randomly selected person has an IQ

1) between 100 and 115
2) More than 135.
3) Less than 70.

EX: A survey was done to measure the haemoglobin (Hb) levels among a group of pregnant women attending an ante-natal clinic. 10000 women were screened and the mean Hb was found to be 10.5 gm%.The standard deviation was 0.5. Compute:

1) Number of women having Hb level between 10 and 11 gm%
2) Number of women having Hb level between 9.5 and 11.5 gm%
3) Number of women having Hb level above 10.5 gm%.
4) Number of women having Hb level below 9 gm%.
5) Number of women having Hb level between 11 gm% and 11.5 gm%.
6) Number of women having Hb level below 9 gm% and above 12 gm%.
7) What is the probability of selecting a pregnant woman with Hb levels below 10 gm%?

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

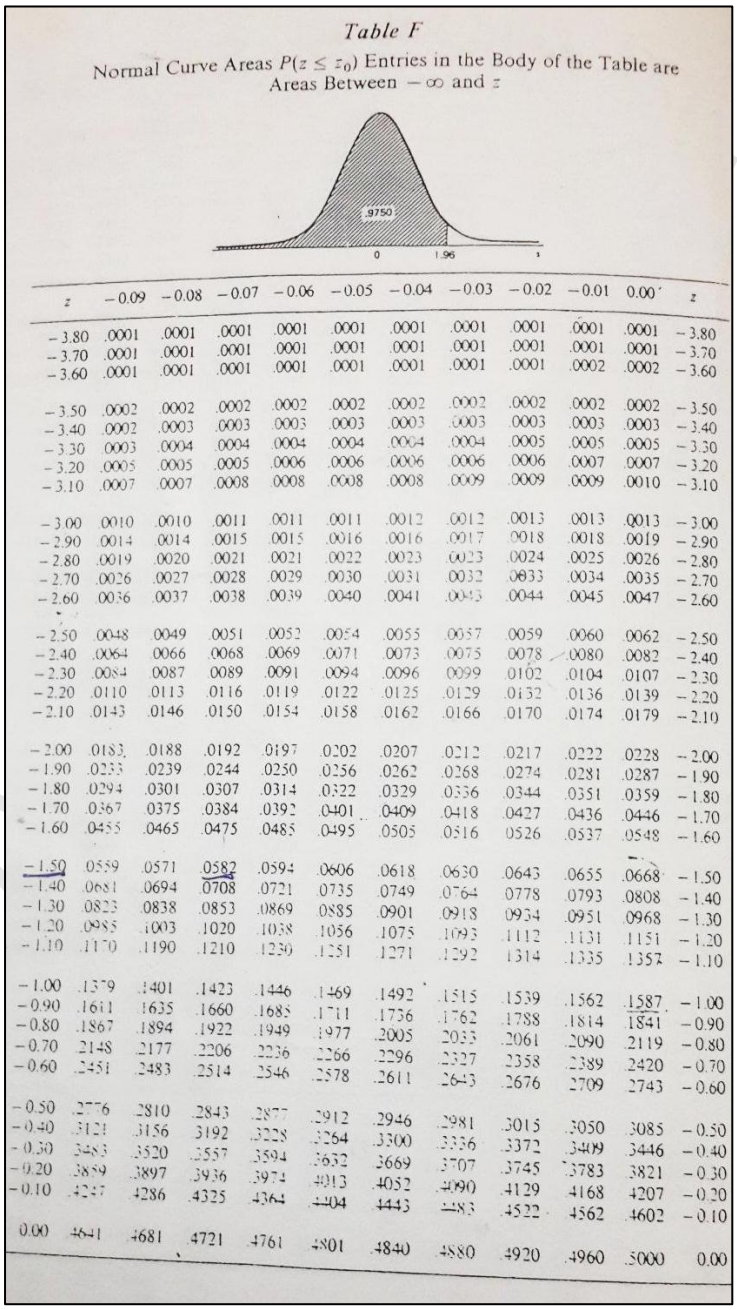

Table F

Normal Curve Areas $P(z \le z_0)$ Entries in the Body of the Table are Areas Between $-\infty$ and z

z	−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	0.00	z
−3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.80
−3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.70
−3.60	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	−3.60
−3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	−3.50
−3.40	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	−3.40
−3.30	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	−3.30
−3.20	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	−3.20
−3.10	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	−3.10
−3.00	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	−3.00
−2.90	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	−2.90
−2.80	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	−2.80
−2.70	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	−2.70
−2.60	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	−2.60
−2.50	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	−2.50
−2.40	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	−2.40
−2.30	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	−2.30
−2.20	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	−2.20
−2.10	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	−2.10
−2.00	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	−2.00
−1.90	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	−1.90
−1.80	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	−1.80
−1.70	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	−1.70
−1.60	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	−1.60
−1.50	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	−1.50
−1.40	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	−1.40
−1.30	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	−1.30
−1.20	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	−1.20
−1.10	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	−1.10
−1.00	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	−1.00
−0.90	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	−0.90
−0.80	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	−0.80
−0.70	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420	−0.70
−0.60	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	−0.60
−0.50	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	−0.50
−0.40	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	−0.40
−0.30	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	−0.30
−0.20	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	−0.20
−0.10	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	−0.10
0.00	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	0.00