Angle Modulation

Introduction

- Angle modulation is the process by which the angle (frequency or phase) of the carrier signal is changed in accordance with the instantaneous amplitude of modulating or message signal.
- The amplitude of the carrier wave is kept constant.
- classified into two types such as
  - ***Frequency modulation (FM)***
  - ***Phase modulation (PM)***
- Used for :
  - Commercial radio broadcasting
  - Television sound transmission
  - Two way mobile radio
  - Cellular radio
  - Microwave and satellite communication system

REPRESENTATION OF FM AND PM SIGNALS

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \text{phase})$$

Amplitude

Frequency

Phase

Angle modulation is a variation of one of these two parameters.

Consider again the general carrier $v_c(t) = V_c \cos(\omega_c t + \phi_c)$

$(\omega_c t + \phi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

a) By varying the frequency, ω_c – **Frequency Modulation.**

b) By varying the phase, ϕ_c – **Phase Modulation**

Phase Modulation

- When the phase of the carrier varies as per amplitude of modulating signal, then it is called phase modulation (PM).
- Amplitude of the modulated carrier remains constant in both modulation systems.

REPRESENTATION OF PM SIGNAL

- An angle-modulated signal

$$v_c = A_c \cos\left(2\pi f_c t + \phi(t)\right)$$

  - where f_c denotes the carrier frequency and $\phi(t)$ denotes a time-varying phase
  - If $m(t)$ is the message signal, then in a PM system, the phase is proportional to the message,

$$\phi(t) = k_p m(t)$$

  - where k_p represents the ***phase sensitivity*** of the modulator , expressed in ***radian per volt***.

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

- Where ϕ = phase angle of carrier signal. It is changed in accordance with the amplitude of the message signal;
- i.e.

$$\phi = K_p V_m(t) = K_p V_m \cos\omega_m t$$

- After phase modulation the instantaneous voltage will be

$$v_{pm}(t) = V_C \cos(\omega_C t + K_p V_m \cos \omega_m t)$$

or

$$v_{pm}(t) = V_C \cos(\omega_C t + m_p \cos \omega_m t)$$

- Where $\mathbf{m_p}$ = ***Modulation index*** of phase modulation

Frequency Modulation

- During the process of frequency modulations the frequency of carrier signal is changed in accordance with the instantaneous amplitude of message signal.

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal $$v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$$

Where the frequency deviation will depend on $m(t)$.

Given that the carrier frequency will change we may write for an instantaneous carrier signal

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\phi_i)$$

where ϕ_i is the instantaneous angle = $\omega_i t = 2\pi f_i t$

and f_i is the instantaneous frequency

Since $\phi_i = 2\pi f_i t$ then $\frac{d\phi_i}{dt} = 2\pi f_i$ or $f_i = \frac{1}{2\pi}\frac{d\phi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

Let message signal: $m(t) = V_m \cos \omega_m t$

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$\omega_i = \omega_c + k_f\, m(t) = \omega_C + k_f\, V_m \cos \omega_m t$$

k_f : represents the ***frequency sensitivity*** of the modulator , expressed in ***Hertz per volt***.

$k_f\, V_m = \Delta f$ The quantity Δf is called the *frequency deviation*, representing the maximum departure of the instantaneous frequency of the FM signal form the carrier frequency f_c.

The amount of change in carrier frequency produced by the modulating signal is known as ***frequency deviation.***

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\phi_i}{dt}$$

Hence, we have $\frac{1}{2\pi}\frac{d\phi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$, *i.e.* $\frac{d\phi_i}{dt} = 2\pi f_c + 2\pi\Delta f_c \cos(\omega_m t)$

After integration *i.e.* $\int(\omega_c + 2\pi\Delta f_c \cos(\omega_m t))dt$

$$\phi_i = \omega_c t + \frac{2\pi\Delta f_c \sin(\omega_m t)}{\omega_m} \Longrightarrow \phi_i = \omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = V_c \cos(\phi_i)$

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)\right)$$

The ratio $\frac{\Delta f_c}{f_m}$ is called the **Modulation Index** denoted by β *i.e.*

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Time domain representation

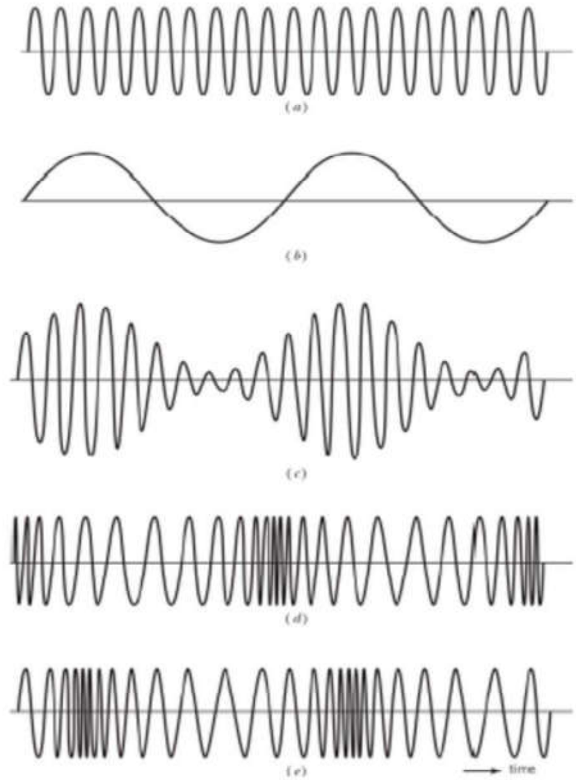
a) Carrier wave

b) Sinusoidal modulating signal

c) Amplitude-modulated signal

d) Phase-modulated signal

e) Frequency-modulated signal



Frequency Deviation

- **Inst. frequency has upper and lower bounds given by**

$$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$

where

$$\Delta f = \textit{frequency deviation} = k_f V_m$$

then

$$f_i\big|_{\max} = f_c + \Delta f$$

$$f_i\big|_{\min} = f_c - \Delta f$$

FM Modulation index

- **The equivalent of AM modulation index is β which is also called *deviation ratio*. It quantifies how much carrier frequency swings relative to message bandwidth**

$$\beta = \frac{\Delta f}{\underbrace{f_m}_{tone}}$$

Example:

- **A 100 MHz FM carrier is modulated by an audio tone causing 20 KHz frequency deviation. Determine the carrier swing and highest and lowest carrier frequencies**

$$\Delta f = 20\,KHz$$
$$frequency\ \ swing\ = 2\Delta f = 40\,KHz$$
$$frequency\ \ range:$$
$$f_{high} = 100\,MHz + 20\,KHz = 100.02\,MHz$$
$$f_{low} = 100\,MHz - 20\,KHz = 99.98\,MHz$$

Example:

- **What is the modulation index (or deviation ratio) of an FM signal with carrier swing of 150 KHz when the modulating signal is 15 KHz?**

$$\Delta f = \frac{150}{2} = 75KHz$$
$$\beta = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process – *i.e.* the principle of superposition does not apply. The FM signal for a message $m(t)$ as a band of signals is very complex. Hence, $m(t)$ is usually considered as a 'single tone modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

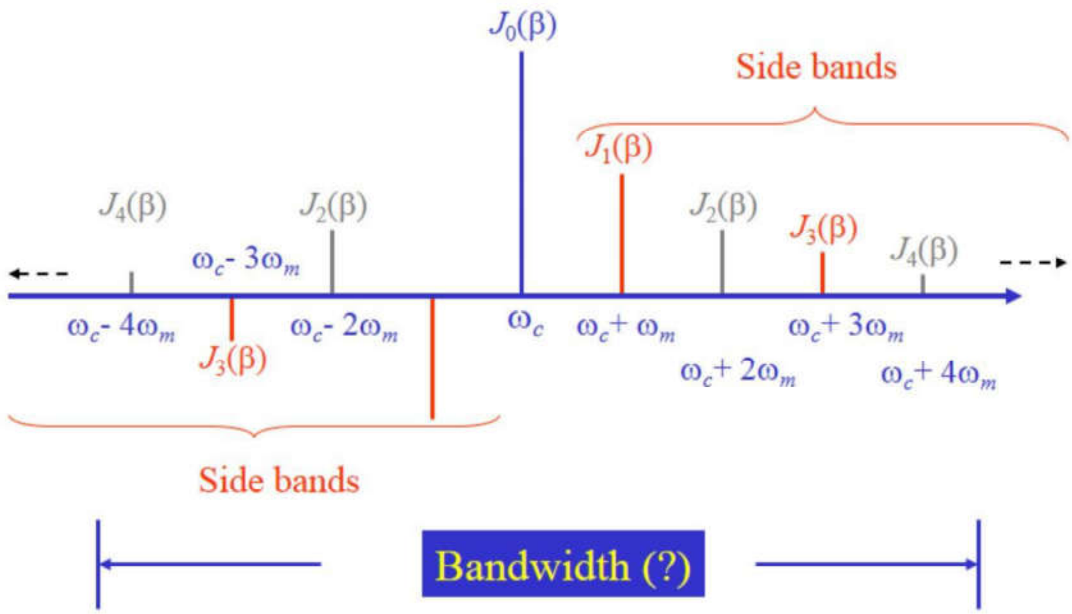
The equation $v_s(t) = V_c \cos(\omega_c t + \beta \sin(\omega_m t))$ may be expressed as Bessel series (Bessel functions)

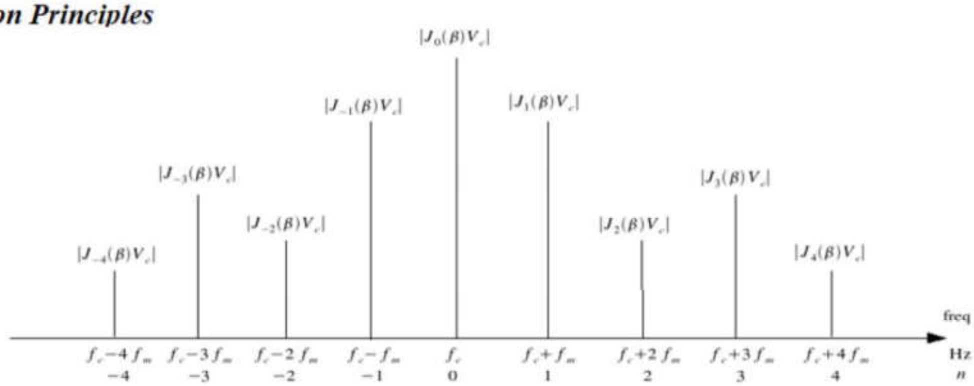
$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ are Bessel functions of the first kind. Expanding the equation for a few terms we have:

$$v_s(t) = \underbrace{V_c J_0(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c)}_{f_c}t + \underbrace{V_c J_1(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + \omega_m)}_{f_c+f_m}t + \underbrace{V_c J_{-1}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - \omega_m)}_{f_c-f_m}t$$
$$+ \underbrace{V_c J_2(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + 2\omega_m)}_{f_c+2f_m}t + \underbrace{V_c J_{-2}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - 2\omega_m)}_{f_c-2f_m}t + \cdots$$

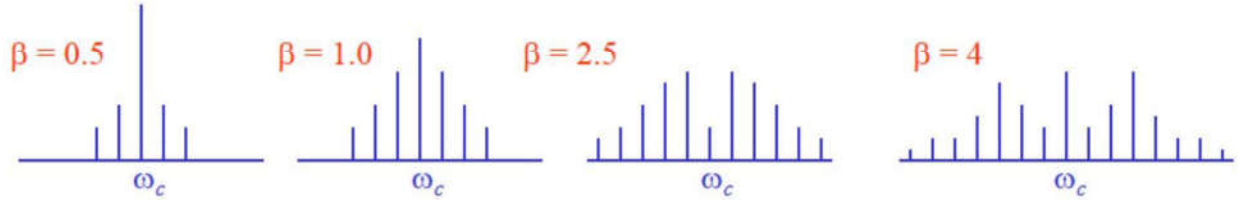
FM Spectrum





The amplitudes drawn are completely arbitrary, since we have not found any value for $J_n(\beta)$ – this sketch is only to illustrate the spectrum.

- The number of side bands with significant amplitude depend on β see below



Generation and transmission of pure FM requires infinite bandwidth, whether or not the modulating signal is band limited. However practical FM systems do have a finite bandwidth with quite well performance.

FM Spectrum – Bessel Coefficients.

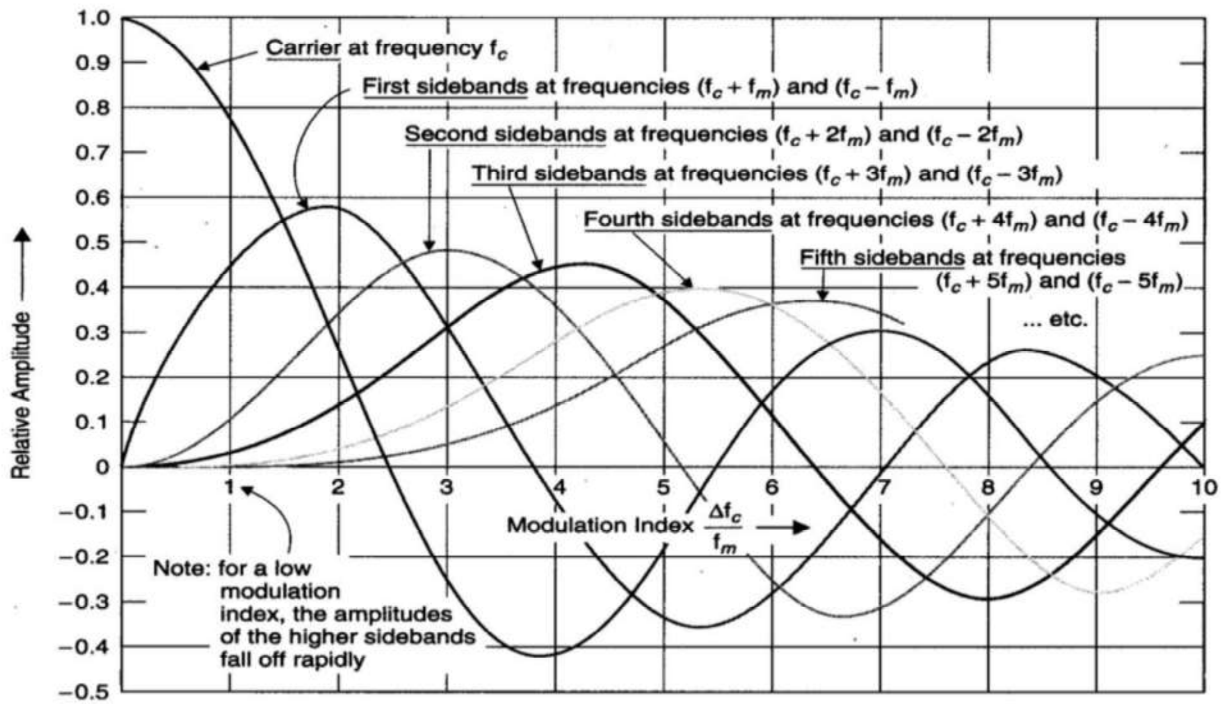
The FM signal spectrum may be determined from

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

The values for the Bessel coefficients, $J_n(\beta)$ may be found from graphs or, preferably, tables of 'Bessel functions of the first kind'.

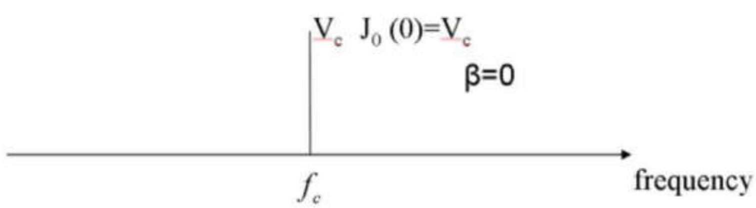
Bessel Functions of the First Kind, $J_n(m)$ for some value of modulation index

Modulation Index	Carrier	Side Frequency Pairs													
β	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01



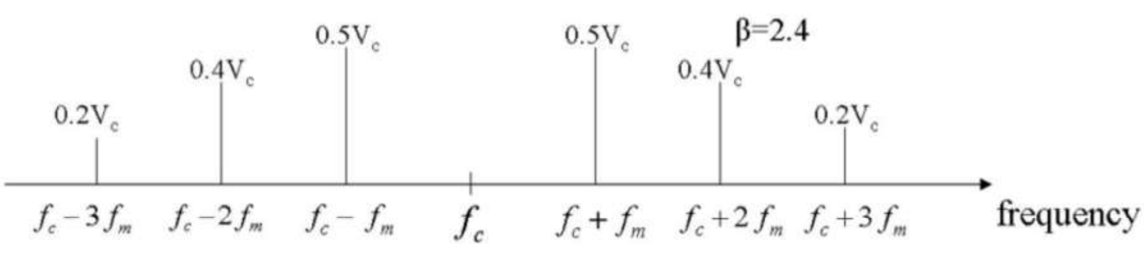
Examples from the graph

β = 0: When β **= 0 the carrier is unmodulated and $J_0(0)$ = 1, all other $J_n(0)$ = 0, *i.e.***



β = 2.4: From the graph (approximately)

$J_0(2.4) = 0$, $J_1(2.4) = 0.5$, $J_2(2.4) = 0.45$ and $J_3(2.4) = 0.2$



Significant Sidebands – Spectrum.

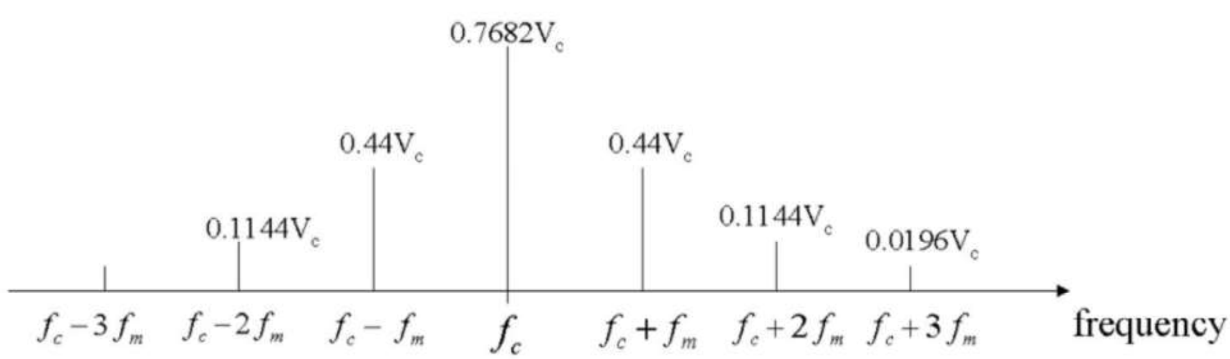
As may be seen from the table of Bessel functions, for values of n above a certain value, the values of $J_n(\beta)$ become progressively smaller. In FM the sidebands are considered to be significant if $J_n(\beta) \geq 0.01$ (1%).

Although the bandwidth of an FM signal is infinite, components with amplitudes $V_c J_n(\beta)$, for which $J_n(\beta) < 0.01$ are deemed to be insignificant and may be ignored.

Example: A message signal with a frequency f_m Hz modulates a carrier f_c to produce FM with a modulation index $\beta = 1$. Sketch the spectrum.

n	$J_n(1)$	Amplitude	Frequency
0	0.7652	$0.7652V_c$	f_c
1	0.4400	$0.44V_c$	f_c+f_m $f_c - f_m$
2	0.1149	$0.1149V_c$	f_c+2f_m $f_c - 2f_m$
3	0.0196	$0.0196V_c$	f_c+3f_m f_c -3 f_m
4	0.0025	*Insignificant*	
5	0.0002	*Insignificant*	

Significant Sidebands – Spectrum.



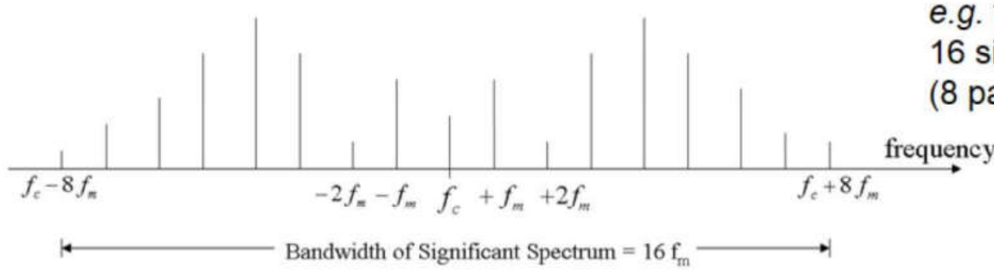
Bandwidth of Significant Spectrum = 6 f_m

As shown, the bandwidth of the spectrum containing significant components is $6f_m$, for $\beta = 1$.

Significant Sidebands – Spectrum.

The table below shows the number of significant sidebands for various modulation indices (β) and the associated spectral bandwidth.

β	No of sidebands ≥ 1% of unmodulated carrier	Bandwidth
0.1	2	$2f_m$
0.3	4	$4f_m$
0.5	4	$4f_m$
1.0	6	$6f_m$
2.0	8	$8f_m$
5.0	16	$16f_m$
10.0	28	$28f_m$



e.g. for β = 5, 16 sidebands (8 pairs).

FM Bandwidth

- **Theoretically, the generation and transmission of FM requires infinite bandwidth. Practically, FM system have finite bandwidth and they perform well.**
- **The value of modulation index determine the number of sidebands that have the significant relative amplitudes.**

- The commonly rule used to determine the bandwidth is:
  - Sideband amplitudes < 1% of the un-modulated carrier can be ignored. Thus $|J_n(\beta)| > 0.01$
- **If n is the number of sideband pairs, and line of frequency spectrum are spaced by fm, thus, the bandwidth is:**

$$B_{fm} = 2nf_m$$

Estimation of transmission bandwidth:

- Assume β is large and n is approximate $\beta + 2$; thus

$B_{fm}=2(\beta + 2) * f_m$ → $2(\frac{\Delta f}{f_m} + 2) f_m$

$$B_{fm} = 2(\Delta f + f_m)........(1)$$

(1) is called Carson's rule

Carson's Rule for FM Bandwidth.

An approximation for the bandwidth of an FM signal is given by
BW = 2(Maximum frequency deviation + highest modulated frequency)

$$\text{Bandwidth} = 2(\Delta f_c + f_m)$$ **Carson's Rule**

Example:

Determine bandwidth with table of bessel functions

Calculate the bandwidth occupied by a FM signal with a modulation index of 2 and a highest modulating frequency of 2.5 kHz.

Solution:

Referring to the table, we can see that this produces six significant pairs of sidebands. The bandwidth can then be determined with the simple formula

$$B.W = 2Nf_{max}$$

where N is the number of significant sidebands.

Using the example above and assuming a highest modulating frequency of 2.5 kHz, the bandwidth of the FM signal is

$$B.W. = 2 \times 6 \times 2.5$$
$$= 30kHz$$

Example:

Assuming a maximum frequency deviation of 5 kHz and a maximum modulating frequency of 2.5 kHz, the bandwidth would be

$$
\begin{aligned}
B.W. &= 2(2.5kHz + 5kHz) \\
&= 2 \times 7.5kHz \\
&= 15kHz
\end{aligned}
$$

Comparing the bandwidth with that computed in the preceding example, you can see that Carson's rule gives a smaller bandwidth.

Narrowband and Wideband FM

Narrowband FM NBFM

From the graph/table of Bessel functions it may be seen that for small β, ($\beta \leq 0.3$) there is only the carrier and 2 significant sidebands, *i.e.* BW = 2*fm*.

FM with $\beta \leq 0.3$ is referred to as **narrowband FM** (NBFM) (Note, the bandwidth is the same as DSBAM).

❖ This is just like AM. No advantage here.

Wideband FM WBFM

For $\beta > 0.3$ there are more than 2 significant sidebands. As β increases the number of sidebands increases. This is referred to as **wideband FM** (WBFM).

❖ This is what we have for a true FM signal

Comparison NBFM & WBFM

S.No	WBFM	NBFM
I.	Modulating index is greater than1	Modulation index is less than 1
II.	Frequency deviation =75 KHz.	Frequency deviation 5 Khz.
III.	Modulating frequency range from 30Hz-15kHz	Modulation frequency =3 Khz
IV.	Bandwidth 15 times NBFM.	Bandwidth = 2Fm
V.	Noise is more suppressed.	Less suppressing of noise
	Use: Entertainment and broadcasting	Use: Mobile communication.

FM Power Distribution

- As seen in Bessel function table, it shows that as the sideband relative amplitude increases, the carrier amplitude,J_0 decreases.
- This is because, in FM, the total transmitted power is always constant and the total average power is equal to the unmodulated carrier power, that is the amplitude of the FM remains constant whether or not it is modulated.
- In effect, in FM, the total power that is originally in the carrier is redistributed between all components of the spectrum, in an amount determined by the modulation index, β, and the corresponding Bessel functions.
- At certain value of modulation index, the carrier component goes to zero, where in this condition, the power is carried by the sidebands only.

Power in FM Signals.

From the equation for FM $v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$

we see that the peak value of the components is $V_c J_n(\beta)$ for the n^{th} component.

Single normalised average power = $\left(\frac{V_{pk}}{\sqrt{2}}\right)^2 = (V_{RMS})^2$ then the n^{th} component is

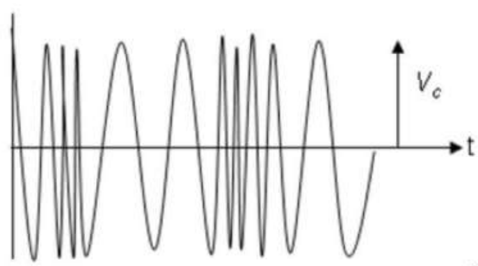
$$\left(\frac{V_c J_n(\beta)}{\sqrt{2}}\right)^2 = \frac{(V_c J_n(\beta))^2}{2}$$

Hence, the total power in the infinite spectrum is

Total power $P_T = \sum_{n=-\infty}^{\infty} \frac{(V_c J_n(\beta))^2}{2}$

By this method we would need to carry out an infinite number of calculations to find P_T. But, considering the waveform, the peak value is V_c, which is constant.

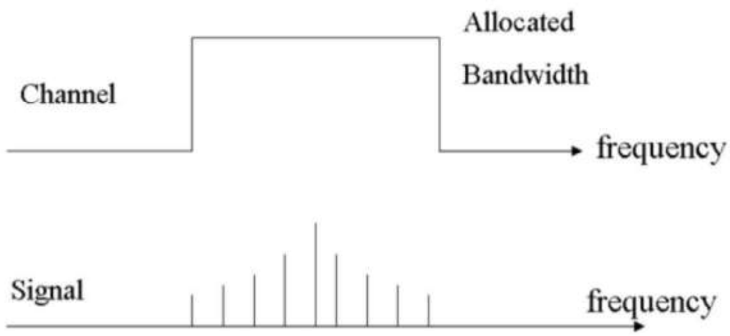
Since we know that the RMS value of a sine wave is $\frac{V_c}{\sqrt{2}}$



and power = $(V_{RMS})^2$ then we may deduce that $P_T = \left(\frac{V_c}{\sqrt{2}}\right)^2 = \frac{V_c^2}{2} = \sum_{n=-\infty}^{\infty} \frac{(V_c J_n(\beta))^2}{2}$

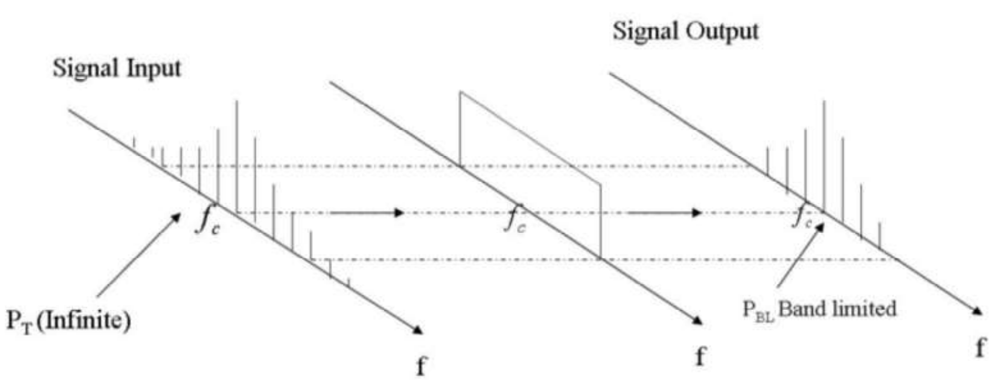
Hence, if we know V_c for the FM signal, we can find the total power P_T for the infinite spectrum with a simple calculation.

Now consider – if we generate an FM signal, it will contain an infinite number of sidebands. However, if we wish to transfer this signal, *e.g.* over a radio or cable, this implies that we require an infinite bandwidth channel. Even if there was an infinite channel bandwidth it would not all be allocated to one user. Only a limited bandwidth is available for any particular signal. Thus we have to make the signal spectrum fit into the available channel bandwidth. We can think of the signal spectrum as a 'train' and the channel bandwidth as a tunnel – obviously we make the train slightly less wider than the tunnel if we can.



However, many signals (*e.g.* FM, square waves, digital signals) contain an infinite number of components. If we transfer such a signal via a limited channel bandwidth, we will lose some of the components and the output signal will be distorted. If we put an infinitely wide train through a tunnel, the train would come out distorted, the question is how much distortion can be tolerated?

Generally speaking, spectral components decrease in amplitude as we move away from the spectrum 'centre'.



In general distortion may be defined as

$$D = \frac{\text{Power in total spectrum - Power in Bandlimited spectrum}}{\text{Power in total spectrum}}$$

$$D = \frac{P_T - P_{BL}}{P_T}$$

With reference to FM the minimum channel bandwidth required would be just wide enough to pass the spectrum of significant components. For a bandlimited FM spectrum, let *a* = the number of sideband pairs, *e.g.* for $\beta = 5$, $a = 8$ pairs (16 components). Hence, power in the bandlimited spectrum P_{BL} is

$$P_{BL} = \sum_{n=-a}^{a} \frac{(V_c J_n(\beta))^2}{2} = \frac{V_c^{\,2}}{2} \sum_{n=-a}^{a} (J_n(\beta))^2$$ = carrier power + sideband powers.

Since $P_T = \dfrac{V_c^2}{2}$

Distortion $$D = \frac{\frac{V_c^2}{2} - \frac{V_c^2}{2}\sum_{n=-a}^{a}(J_n(\beta))^2}{V_c^2/2} = 1 - \sum_{n=-a}^{a}(J_n(\beta))^2$$

Also, it is easily seen that the ratio

$$\frac{\text{Power in Bandlimited spectrum}}{\text{Power in total spectrum}} = \frac{P_{BL}}{P_T} = \sum_{n=-a}^{a}(J_n(\beta))^2$$ = 1 – Distortion

i.e. proportion p_f power in bandlimited spectrum to total power = $\sum_{n=-a}^{a}(J_n(\beta))^2$

Example

Consider NBFM, with β = 0.2. Let V_c = 10 volts. The total power in the infinite spectrum $\frac{V_c^2}{2}$ = 50 Watts,

From the table – the significant components are

n	$J_n(0.2)$	$Amp = V_c J_n(0.2)$	$Power = \frac{(Amp)^2}{2}$
0	0.9900	9.90	49.005
1	0.0995	0.995	0.4950125
			P_{BL} = 49.5 Watts

i.e. the carrier + 2 sidebands contain $\frac{49.5}{50} = 0.99$ or 99% of the total power

Distortion = $\frac{P_T - P_{BL}}{P_T} = \frac{50 - 49.5}{50} = 0.01$ or 1%.

Actually, we don't need to know V_c, *i.e.* alternatively

Distortion = $1 - \sum_{n=-1}^{1} (J_n(0.2))^2$ (a = 1) → D $= 1 - (0.99)^2 - (0.0995)^2 = 0.01$

$$\frac{P_{BL}}{P_T} = \sum_{n=-1}^{1} (J_n(\beta))^2 = 1 - D = 0.99$$

FM SIGNAL GENERATION

They are two basic methods of generating frequency-Modulated signals

- Direct Method
- Indirect Method

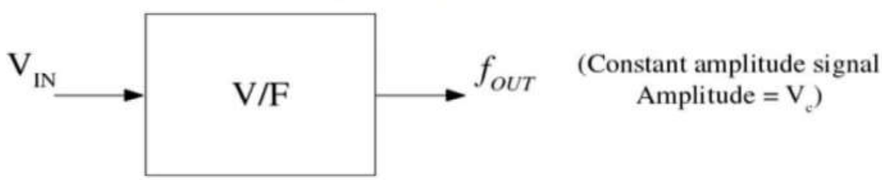
DIRECT METHOD FM GEBERATION

$$f_i = f_C + k_f m(t)$$

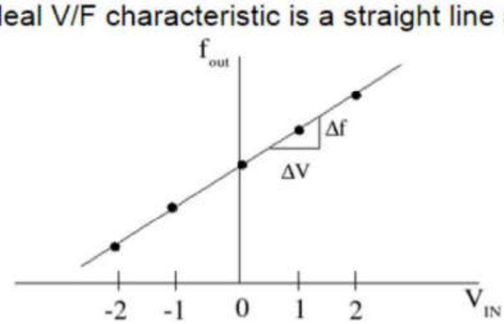
In a direct FM system the instantaneous frequency is directly varied with the information signal. To vary the frequency of the carrier is to use an Oscillator whose resonant frequency is determined by components that can be varied. The oscillator frequency is thus changed by the modulating signal amplitude.

- Straight forward method, requires a VCO whose oscillation frequency has linear dependence on applied voltage.
- Advantage: large frequency deviation
- Disadvantage: the carrier frequency tends to drift and must be stabilized.
- Example circuit:
  - Reactance modulator
  - Varactor diode

In these devices (V/F or VCO), the output frequency is dependent on the input voltage amplitude.



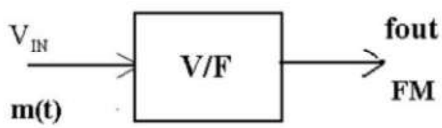
Apply V_{IN} , *e.g.* 0 Volts, +1 Volts, +2 Volts, -1 Volts, -2 Volts, ... and measure the frequency output for each V_{IN} . The ideal V/F characteristic is a straight line as shown below.

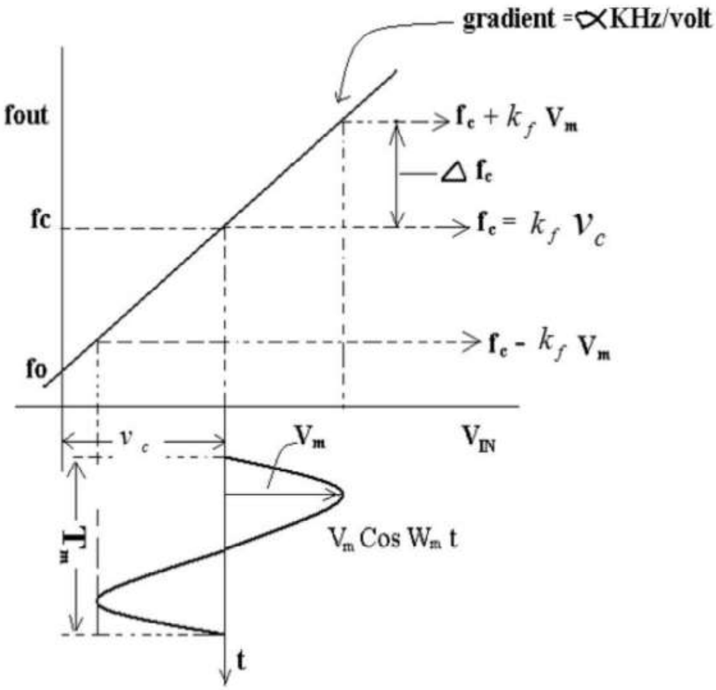


f_c, the frequency output when the input is zero is called the undeviated or nominal carrier frequency.

The gradient of the characteristic $\frac{\Delta f}{\Delta V}$ is called the **Frequency sensitivity of of the modulator** , denoted by k_f (Hertz per Volt).

Consider now, an analogue message input, $m(t) = V_m \cos(\omega_m t)$





As the input *m*(*t*) varies from

$$+V_m \rightarrow 0 \rightarrow -V_m$$

the output frequency will vary from a maximum, through f_c, to a minimum frequency.

For a straight line, $y = c + mx$, where c = value of y when $x = 0$, m = gradient, hence we may say

$$f_{\text{OUT}} = f_c + k_f V_{\text{IN}}$$

and when $V_{IN} = m(t)$ $f_{\text{OUT}} = f_c + k_f m(t)$,*i.e.* the deviation depends on *m*(*t*).

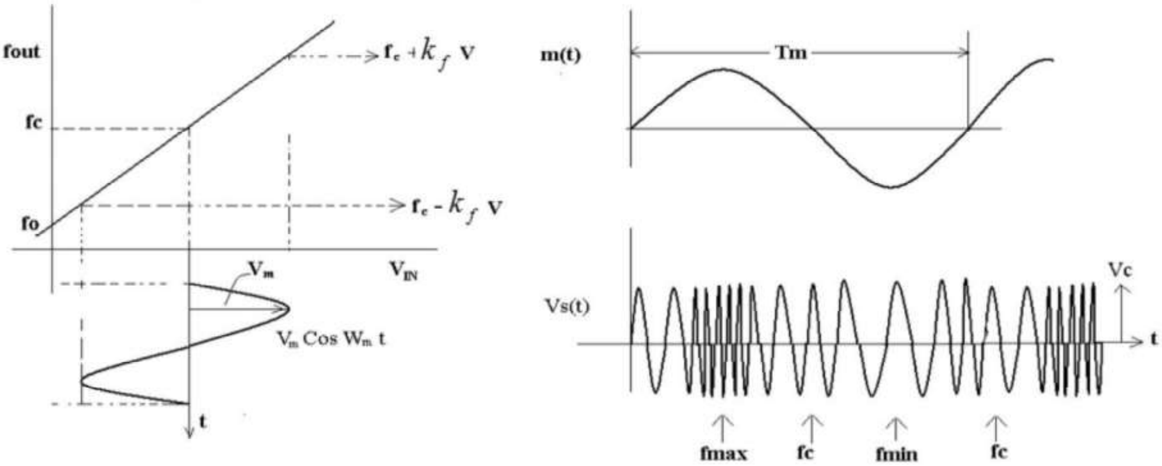
Considering that maximum and minimum input amplitudes are $+V_m$ and $-V_m$ respectively, then

$$f_{\max} = f_c + k_f V_m$$
$$f_{\min} = f_c - k_f V_m$$

on the diagram on the previous slide.

The peak-to-peak deviation is $f_{max} - f_{min}$, but more importantly for FM the peak deviation Δf_c is

Peak Deviation, $\Delta f_c = k_f V_m$ Hence, **Modulation Index**, $\beta = \frac{\Delta f_c}{f_m} = \frac{k_f V_m}{f_m}$



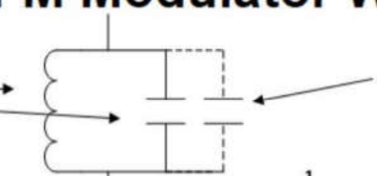
The output frequency varies 'gradually' from f_c to ($f_c + k_f\ V_m$), through f_c to ($f_c - k_f\ V_m$) *etc.*

In general, $m(t)$ will be a 'band of signals', *i.e.* it will contain amplitude and frequency variations. Both amplitude and frequency change in $m(t)$ at the input are translated to (just) frequency changes in the FM output signal, *i.e.* the amplitude of the output FM signal is constant.

Amplitude changes at the input are translated to deviation from the carrier at the output. The larger the amplitude, the greater the deviation.

How FM Modulator Works?

Frequency of resonance depends on the value of inductance and capacitance

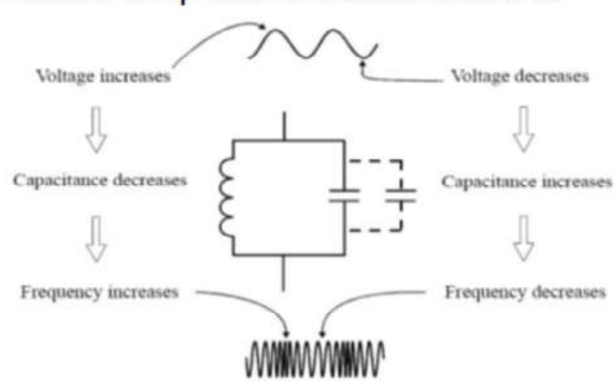


Additional capacitance will increase the capacitance, thus reduce the resonance frequency

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

For FM, we want the frequency increase/decrease wrt to amplitude of modulating signal. How??

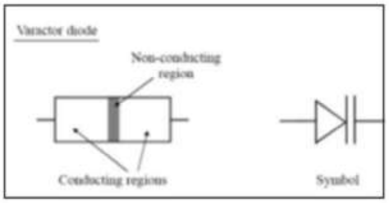
- The tuned circuit is part of the oscillator used to generate the carrier freq so, if the capacitance changes, then so will the carrier freq. This is demonstrated in Figure below.

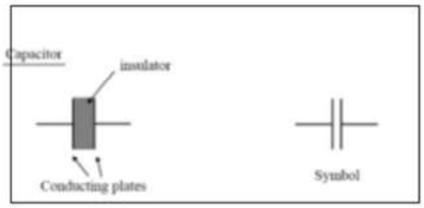


- ❑ To produce a freq modulated carrier, it is needed to find a way of making the info signal increase and decrease the size of the capacitance and hence control the carrier freq.
- ❑ How to achieve? – using a device called Varactor Diode and then by using a transistor.

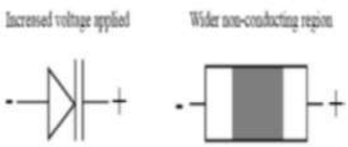
Varactor Diode

- Varactor diode is a semiconductor diode that is designed to behave as a voltage controlled capacitor.
- When a semiconductor diode is reverse biased, no current flows and it consists of two conducting region separated by a non-conducting region.
- This is very similar to the construction of the capacitor.
- Recall, the reverse biased diode has a capacitance of $C_d = \frac{\varepsilon A}{d}$

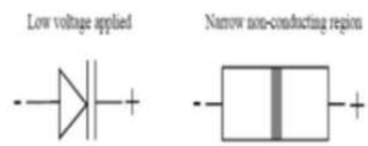




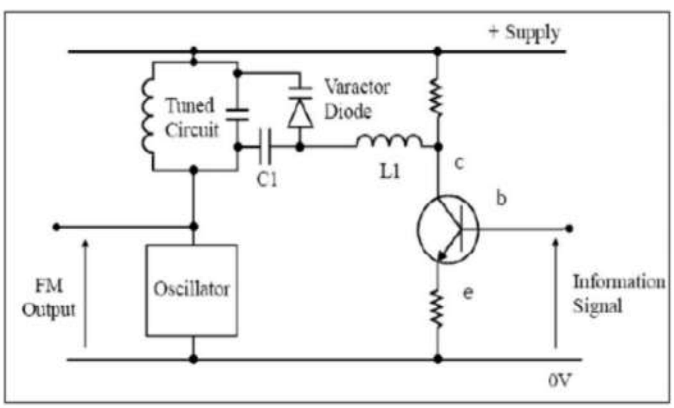
- By increasing the reverse biased voltage, the width of the insulating region can be increased and hence the capacitance value decreased.
- Thus, if the info signal is applied to the varactor diode, the capacitance will Therefore be increased and decreased in sympathy with the incoming signal.



more capacitance. less capacitance.



Varactor Modulator Circuit



- Tuned circuit sets the operating frequency of the oscillator
- C1 is a DC blocking capacitor to provide DC isolation between the oscillator and the collector of the transistor.
- L1 is an RF choke which allows the info signal through to the varactor but blocks the RF signals.

The info signal is applied to the base of the input transistor and appears amplified and inverted at the collector.
- This low freq signal passes through the RF choke (L1) and is applied across the varactor diode.
- The varactor diode changes its capacitance in sympathy with the info signal and therefore changes the total value of the capacitance in the tuned circuit.
- The changing value of capacitance causes the oscillator freq to increase and decrease under the control of the information signal.
- The output is therefore an FM signal.

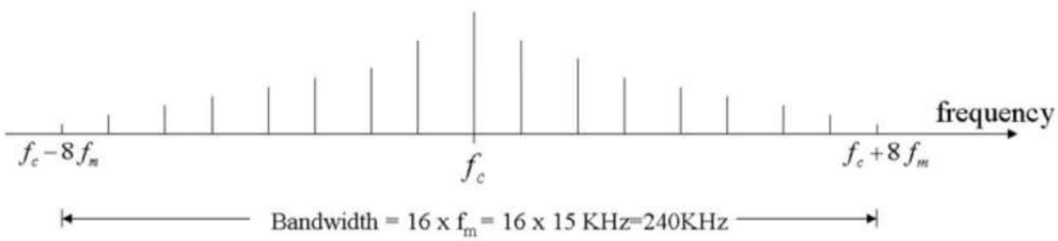
VHF/FM

VHF/FM (Very High Frequency band = 30MHz – 300MHz) radio transmissions, in the band 88MHz to 108MHz have the following parameters:

Max frequency input (*e.g.* music)	15kHz	f_m
Deviation	75kHz	$\Delta f_c = k_f V_m$
Modulation Index β	5	$\beta = \frac{\Delta f_c}{f_m}$

For β = 5 there are 16 sidebands and the FM signal bandwidth is 16*fm* = 16 x 15kHz = 240kHz.

Applying Carson's Rule BW = 2(75+15) = 180kHz.

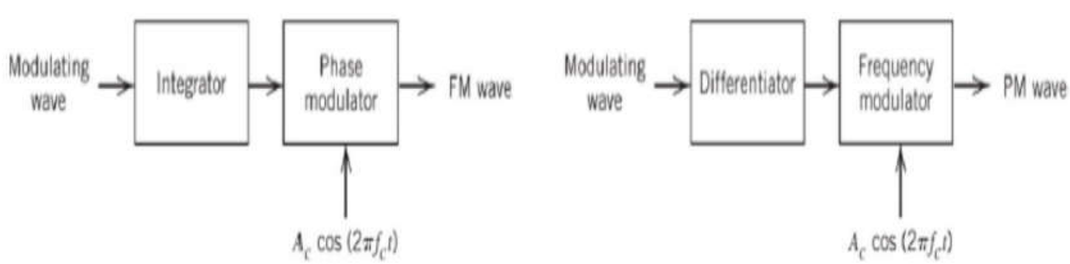


INDIRECT FM GENERATION

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\,d\tau\right]$$

The FM signal can be generated by first integrating $m(t)$ and then using the result as the input to a phase modulator, as in Figure 4.3*a*. Conversely, a PM signal can be generated by first differentiating $m(t)$ and then using the result as the input to a frequency modulator,



Since FM is produced by PM , the later is referred to as indirect FM.

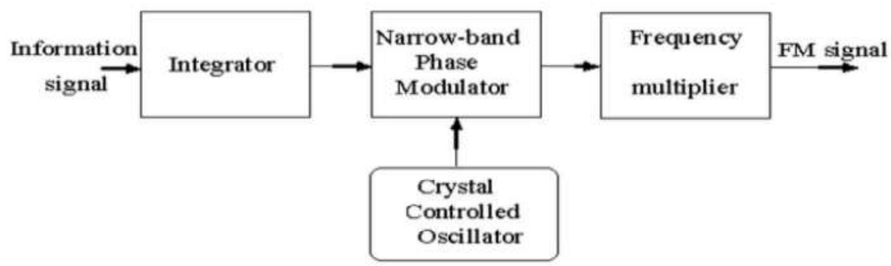
The information signal is first integrated and then used to phase modulate a crystal-controlled oscillator, which provides frequency stability.

In order to minimize the distortion in the phase modulator, the modulation index is kept small, thereby is resulting in a ***narrow-band FM-signal***

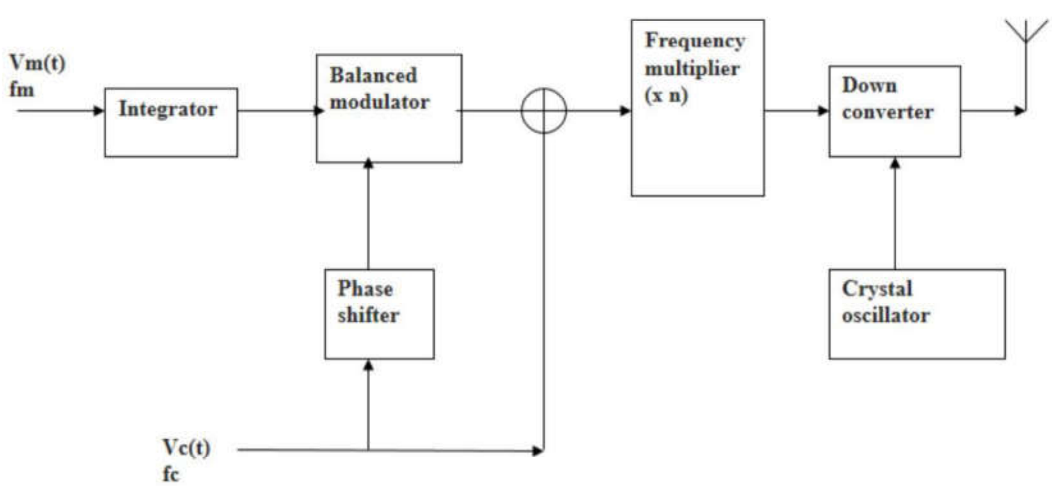
The narrow-band FM signal is multiplied in frequency by means of frequency multiplier so as to produce the desired wide-band FM signal.

The frequency multiplier is used to perform narrow band to wideband conversion.

Block diagram of the indirect method of generating a wide-band FM-signal



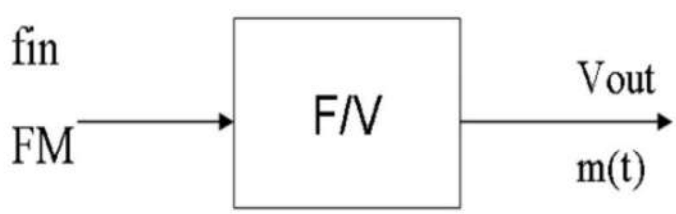
One most popular indirect method is the Armstrong modulator



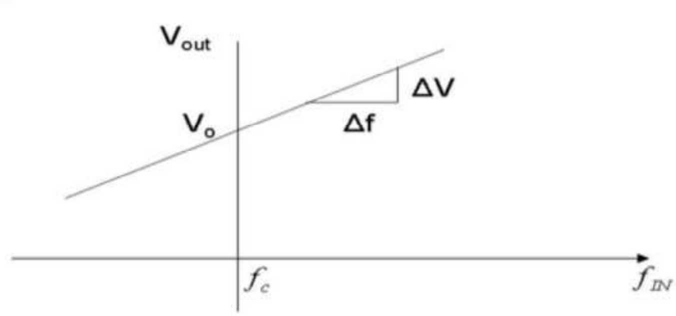
FM Detection/Demodulation

- is a process of getting back or regenerate the original modulating signal from the modulated FM signal.
- It can be achieved by converting the frequency deviation of FM signal to the variation of equivalent voltage.
- The demodulator will produce an output where its instantaneous amplitude is proportional to the instantaneous frequency of the input FM signal.

- An FM demodulator or frequency discriminator is essentially a frequency-to-voltage converter (F/V). An F/V converter may be realised in several ways, including for example, tuned circuits and envelope detectors, phase locked loops *etc.* Demodulators are also called FM discriminators.
- Before considering some specific types, the general concepts for FM demodulation will be presented. An F/V converter produces an output voltage, V_{OUT} which is proportional to the frequency input, f_{IN}.



- In this case f_{IN} is the independent variable and V_{OUT} is the dependent variable (*x* and *y* axes respectively). The ideal characteristic is shown below.



We define V_o as the output when $f_{IN} = f_c$, the nominal input frequency.

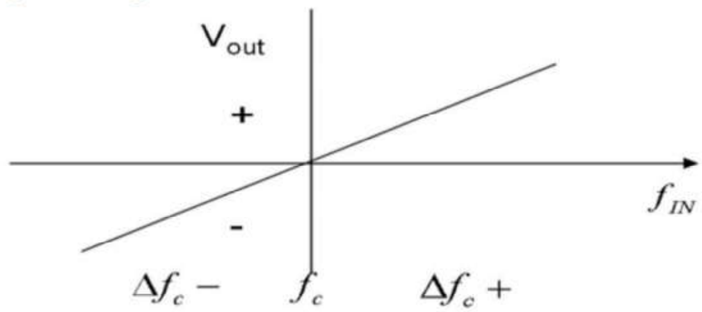
The gradient $\dfrac{\Delta V}{\Delta f}$ is called the voltage conversion factor

i.e. **Gradient = Voltage Conversion Factor**, *K* volts per Hz.

Considering $y = mx + c$ *etc.* then we may say $V_{OUT} = V_0 + Kf_{IN}$ from the frequency modulator, and since $V_0 = V_{OUT}$ when $f_{IN} = f_c$ then we may write

$$V_{OUT} = V_0 + K\,k_f V_{IN}$$

where V_0 represents a DC offset in V_{OUT}. This DC offset may be removed by level shifting or AC coupling, or the F/V may be designed with the characteristic shown next



The important point is that $V_{OUT} = K\,k_f\,V_{IN}$. If $V_{IN} = m(t)$ then the output contains the message signal $m(t)$, and the FM signal has been demodulated.

Often, but not always, a system designed so that $K = \dfrac{1}{k_f}$, so that $Kk_f = 1$ and $V_{OUT} = m(t)$.

Gradient = k_f Hz/Volt

k_f = Frequency sensitivity of the modulator

$f_{OUT} = f_c + k_f V_{IN} = f_{IN}$

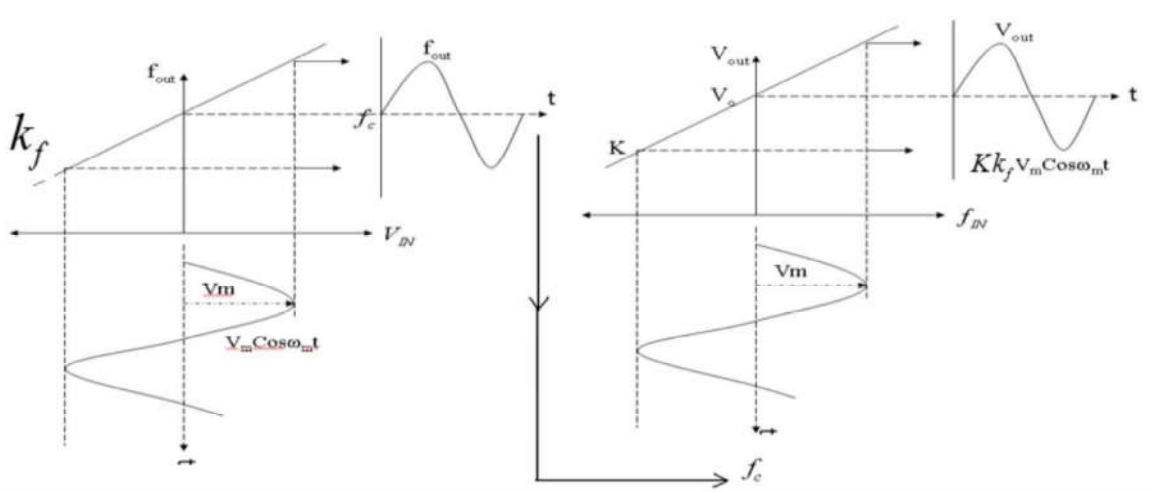
$f_{OUT} = f_c + k_f m(t) = f_{IN}$

Gradient = K Volt / Hz

K = Voltage conversion factor

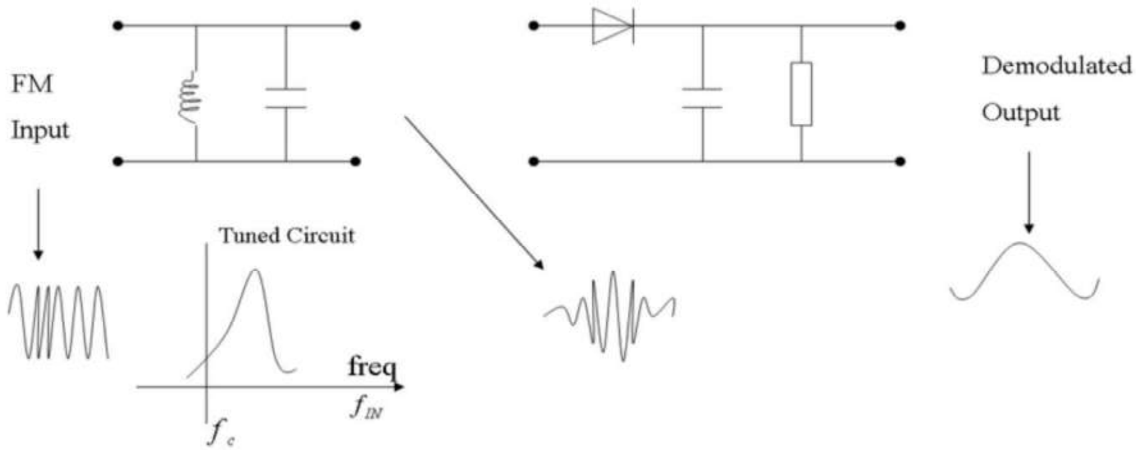
$V_{OUT} = V_0 + Kk_f V_{IN}$

$V_{OUT} = V_0 + Kk_f m(t)$



Detection Methods

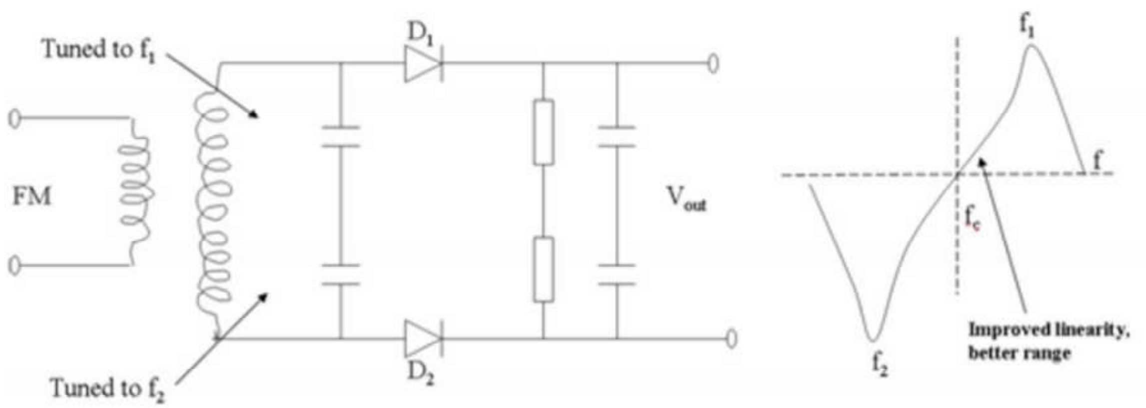
1- Tuned Circuit – One method (used in the early days of FM) is to use the slope of a tuned circuit in conjunction with an envelope detector.



- The tuned circuit is tuned so the f_c, the nominal input frequency, is on the slope, not at the centre of the tuned circuits. As the FM signal deviates about f_c on the tuned circuit slope, the amplitude of the output varies in proportion to the deviation from f_c. Thus the FM signal is effectively converted to AM. This is then envelope detected by the diode *etc* to recover the message signal.

2- Foster-Seeley Discriminator

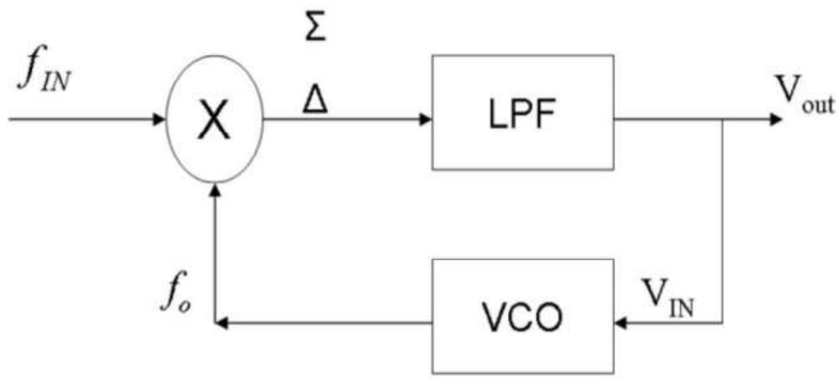
- A better method is to use 2 similar circuits, known as a **Foster-Seeley Discriminator**



This gives the composite characteristics shown. Diode D_2 effectively inverts the f_2 tuned circuit response. This gives the characteristic *'S'* type detector.

3- Phase Locked Loops PLL

- A PLL is a closed loop system which may be used for FM demodulation. A full analytical description is outside the scope of these notes. A brief description is presented. A block diagram for a PLL is shown below.



- Note the similarity with a synchronous demodulator. The loop comprises a multiplier, a low pass filter and VCO (V/F converter as used in a frequency modulator).

- The input f_{IN} is applied to the multiplier and multiplied with the VCO frequency output f_O, to produce $\Sigma = (f_{IN} + f_O)$ and $\Delta = (f_{IN} - f_O)$.
- The low pass filter passes only $(f_{IN} - f_O)$ to give *VOUT* which is proportional to $(f_{IN} - f_O)$.
- If $f_{IN} \approx f_O$ but not equal, $V_{OUT} = V_{IN}$, $\alpha f_{IN} - f_O$ is a low frequency (beat frequency) signal to the VCO.
- This signal, V_{IN}, causes the VCO output frequency f_O to vary and move towards f_{IN}.
- When $f_{IN} = f_O$, V_{IN} $(f_{IN} - f_O)$ is approximately constant (DC) and f_O is held constant, *i.e* locked to f_{IN}.
- As f_{IN} changes, due to deviation in FM, f_O tracks or follows f_{IN}. $V_{OUT} = V_{IN}$ changes to drive f_O to track f_{IN}.
- V_{OUT} is therefore proportional to the deviation and contains the message signal $m(t)$.

Advantages of FM

- Wideband FM gives significant improvement in the SNR at the output of the RX which proportional to the square of modulation index.
- Angle modulation is resistant to propagation-induced selective fading since amplitude variations are unimportant and are removed at the receiver using a limiting circuit.
- Angle modulation is very effective in rejecting interference. (minimizes the effect of noise).
- Angle modulation allows the use of more efficient transmitter power in information.
- Angle modulation is capable of handing a greater dynamic range of modulating signal without distortion than AM.

Disadvantages of FM

- Angle modulation requires a transmission bandwidth much larger than the message signal bandwidth.
- Angle modulation requires more complex and expensive circuits than AM.

Application of FM

- FM is commonly used at VHF radio frequencies for high-fidelity broadcasts of music and speech (FM broadcasting). Normal (analog) TV sound is also broadcast using FM. The type of FM used in broadcast is generally called wide-FM, or W-FM
- A narrowband form is used for voice communications in commercial and radio settings. In two-way radio, narrowband narrow-fm (N-FM) is used to conserve bandwidth. In addition, it is used to send signals into space.

Comparison AM and FM

- Its the SNR can be increased without increasing transmitted power about 25dB higher than in AM
- Certain forms of interference at the receiver are more easily to suppressed, as FM receiver has a limiter which eliminates the amplitude variations and fluctuations.
- The modulation process can take place at a low level power stage in the transmitter, thus a low modulating power is needed.
- Power content is constant and fixed, and there is no waste of power transmitted
- There are guard bands in FM systems allocated by the standardization body, which can reduce interference between the adjacent channels.

Thank you