TIME-VARYING FIELDS AND MAXWELL'S EQUATIONS

FARADAY'S LAW

Faraday professed his belief that if a current could produce a magnetic field, then a magnetic field should be able to produce a current. He wound two separate windings on an iron toroid and placed a galvanometer in one circuit and a battery in the other. Upon closing the battery circuit, he noted a momentary deflection of the galvanometer; a similar deflection in the opposite direction occurred when the battery was disconnected. This, of course, was the first experiment he made involving a *changing* magnetic field, and he followed it with a demonstration that either a *moving* magnetic field or a moving coil could also produce a galvanometer deflection.

Induced Electromotive Force

In terms of fields, we now say that a time-varying magnetic field produces an *electromotive force* (emf) which may establish a current in a suitable closed circuit. An electromotive force is merely a voltage that arises from conductors moving in a magnetic field or from changing magnetic fields, and we shall define it below. Faraday's law is customarily stated as

$$\text{emf} = -\frac{d\Phi}{dt} \quad \text{V} \tag{1}$$

Equation (1) implies a closed path, although not necessarily a closed conducting path; the closed path, for example, might include a capacitor, or it might be a purely imaginary line in space. The magnetic flux is that flux which passes through any and every surface whose perimeter is the closed path, and $d\Phi/dt$ is the time rate of change of this flux.

A nonzero value of $d\Phi/dt$ may result from any of the following situations:

1. A time-changing flux linking a stationary closed path
2. Relative motion between a steady flux and a closed path
3. A combination of the two

If the closed path is that taken by an N-turn filamentary conductor, it is often sufficiently accurate to consider the turns as coincident and let

$$\text{emf} = -N\frac{d\Phi}{dt} \tag{2}$$

where Φ is now interpreted as the flux passing through any one of N coincident paths.

We need to define emf as used in (1) or (2). The emf is obviously a scalar, and (perhaps not so obviously) a dimensional check shows that it is measured in volts. We define the emf as

$$\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} \tag{3}$$

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0 \quad \text{(electrostatics)}$$

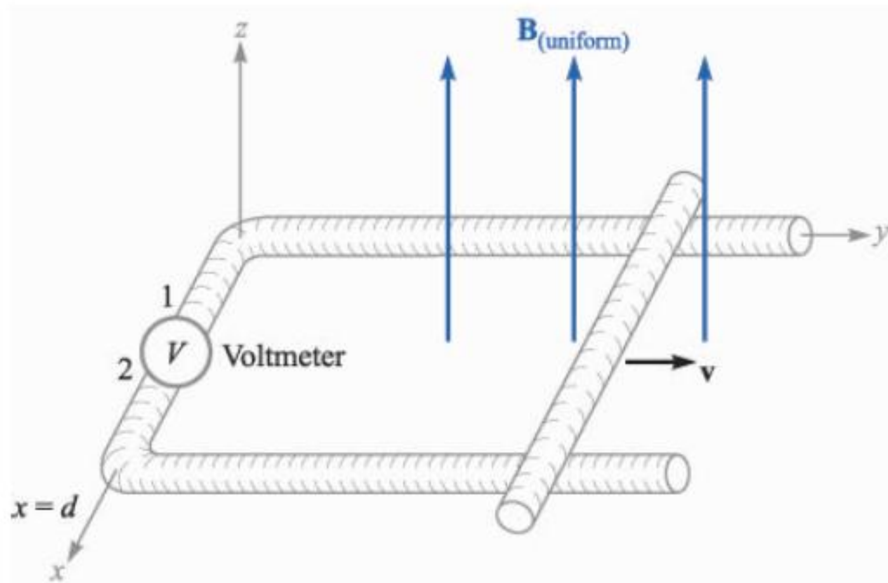
Replacing Φ in (1) by the surface integral of **B**, we have

$$\boxed{\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}} \tag{4}$$

$$\text{emf} = \oint \mathbf{E} \cdot d\mathbf{L} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} \tag{5}$$

$$\oint \mathbf{E} \cdot d\mathbf{L} = 0 \quad \text{(electrostatics)}$$

An example illustrating the application of Faraday's law to the case of a constant magnetic flux density **B** and a moving path. The shorting bar moves to the right with a velocity **v**, and the circuit is completed through the two rails and an extremely small high-resistance voltmeter. The voltmeter reading is $V_{12} = -Bvd$.



$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\phi = \mathbf{B} \cdot \mathbf{S}$$

$$\Phi = Byd \qquad (1)$$

From (1), we obtain

$$\text{emf} = -\frac{d\Phi}{dt} = -B\frac{dy}{dt}d = -Bvd$$

The circular loop conductor shown in Fig. 12-10 lies in the $z = 0$ plane, has a radius of 0.10 m and a resistance of 5.0 Ω. Given $\mathbf{B} = 0.20 \sin 10^3 t \mathbf{a}_z$ (T), determine the current.

$$\phi = \mathbf{B} \cdot \mathbf{S} = 2 \times 10^{-3} \pi \sin 10^3 t \quad \text{(Wb)}$$

$$v = -\frac{d\phi}{dt} = -2\pi \cos 10^3 t \quad \text{(V)}$$

$$i = \frac{v}{R} = -0.4\pi \cos 10^3 t \quad \text{(A)}$$

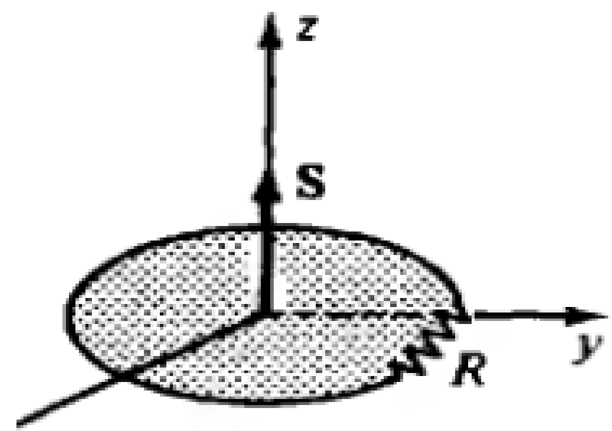


Fig. 12-10

A square coil, 0.60 m on a side, rotates about the x axis at $\omega = 60\pi$ rad/s in a field $\mathbf{B} = 0.80\mathbf{a}_z$ T, as shown in Fig. 12-16(a). Find the induced voltage.

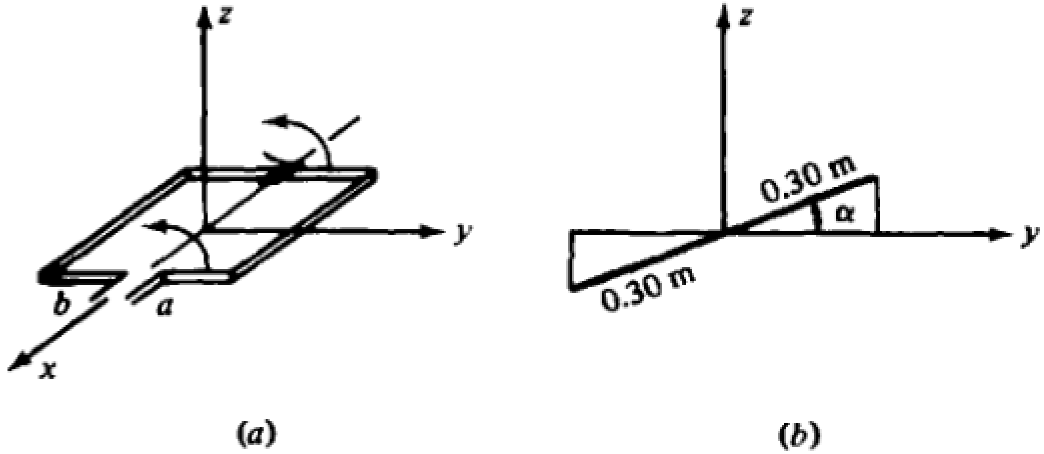


Fig. 12-16

Assuming that the coil is initially in the xy plane,

$$\alpha = \omega t = 60\pi t \text{ (rad)}$$

The projected area on the xy plane becomes [see Fig. 12-16(b)]:

$$A = (0.6)(0.6 \cos 60\pi t) \quad (\text{m}^2)$$

Then $\phi = BA = 0.288 \cos 60\pi t$ (Wb) and

$$v = -\frac{d\phi}{dt} = 54.3 \sin 60\pi t \quad (\text{V})$$